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Activity Theory and e-Course Design: An Experience in Discrete Mathematics for Computer Science

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Abstract

The aim of this article is to present a distance e-learning experience of mathematics in higher education. The course is offered as a remedial program for master's degree students of Computer Science. It was designed to meet the particular needs of the students entering the master's degree program, as a response to the lack of understanding of logical language which was identified in several previous cohorts of students at CENIDET. The course addresses mathematical abilities of comprehensive functional use of logical language as a basic ability to be developed for later successful participation in the Master of Computer Science and also for later use in professional contexts of Computer Engineering. Eighteen students distributed throughout Mexico volunteered to participate under the guidance of one instructor. The techno-pedagogical design of the course is grounded on two theoretical approaches. Content-related instructional decisions are supported by different concepts of the second generation of Activity Theory. The concept of Orienting Basis of an Action was particularly useful to define the skills the students were expected to develop. Instructional decisions related to the participants' interaction are underpinned by Slavin's Team Accelerated Instruction model. We present the course structure in detail and provide some student interaction excerpts in order to illustrate their learning progress.

Keywords

e-learning, instructional design, higher education, discrete mathematics, activity theory, mathematical abilities

Teoría de la actividad y diseño de cursos virtuales: la enseñanza de matemáticas discretas en Ciencias de la Computación

Resumen

El objetivo de este estudio es presentar una experiencia de aprendizaje virtual a distancia en el ámbito de la enseñanza de las matemáticas en educación superior. El curso se ofrece como programa de apoyo para alumnos de un máster de Ciencias de la Computación y está específicamente diseñado para satisfacer las necesidades de los estudiantes que iniciaban dicho programa, particularmente la falta de comprensión del lenguaje lógico detectada en varias promociones anteriores de los alumnos del CENIDET. El curso tiene como objetivo el desarrollo de la habilidad de uso del lenguaje lógico, la cual es básica para cursar con éxito el máster de Ciencias de la Computación, así como para su posterior aplicación en contextos profesionales relacionados con la Ingeniería computacional. Dieciocho estudiantes distribuidos por todo México participaron voluntariamente en el estudio bajo la dirección de un tutor. El diseño tecnopedagógico del curso se basa en dos premisas teóricas. Las decisiones didácticas relacionadas con el contenido se fundamentan en varios conceptos derivados de la segunda generación de la Teoría de la Actividad (TA). El concepto de «base de orientación para la acción» ha sido particularmente útil para definir las habilidades que se esperaba que desarrollaran los estudiantes. Las decisiones didácticas relacionadas con la interacción de los participantes se basan en el modelo de enseñanza acelerada en equipo de Slavin. A continuación se expone detalladamente la estructura del curso y se presentan algunos extractos de la interacción de los estudiantes para ilustrar su proceso de aprendizaje.

Palabras clave

aprendizaje virtual, diseño didáctico, educación superior, matemática discreta, teoría de la actividad, habilidades matemáticas

1. Introduction

Working with formalized or semi-formalized statements is often a challenge for many students of mathematics. A typical coping strategy is to read the statements informally, avoiding mathematical formalism. Unfortunately, an important loss of mathematical knowledge happens by using this strategy. Various researchers have linked these difficulties with: i) the negation of mathematical statements (Antonini, 2001; Durand-Guerrier, 2004); ii) the translation (formalization) of natural-language statements into the formal language of First Order Logic (FOL) (Barker-Plummer, Cox, Dale & Etchemendy, 2008); and iii) the identification of the logical structure of mathematical statements (Selden & Selden, 1996).

In the field of Computer Science (CS), there have been recent proposals to include Formal Methods in the curriculum. Students are now expected to develop skills to read and write formal specifications for professional practice (Boca, Bowen & Duce, 2006). However, although many of them first encounter formalized or semi-formalized mathematics in Discrete Mathematics (DM) courses, their instructors often expect them to master FOL already. Hence, the students face difficulties in understanding and communicating new and complex concepts in their discipline with semi-formalized texts. Consequently, they need specific help to develop the skills to read mathematical texts in different settings. A good presentation of content does not suffice; therefore, remedial courses in higher education must work towards the explicit development of this ability (Merisotis & Phipps, 2000).

This paper describes the use of some elements of the second generation of Activity Theory (AT) to design an online remedial course. Particularly, the concept of Orienting Basis of an Action (OBA) was helpful to provide master's degree students with the necessary support in their learning processes. The online remedial course covers DM Preliminary Concepts (Logic, Set, Relations and Functions) for students entering a Master of CS in Mexico. In the following sections, we shall present the contextual background in detail, the theoretical assumptions and the consequences for instructional design. Some excerpts of the interactions occurring during the course will illustrate the students' learning progress.

2. The institutional context: mathematics teaching in Computer Science

Most DM courses offered to CS students follow a traditional model of mathematics teaching, such as: (1) concept definition; (2) theorem presentation; (3) demonstration; and (4) exercising (see Meyer, 2005, for example). Alternative courses are still the exception. Furthermore, they usually lack the theoretical underpinning of mathematical education (see, for example, Sutner, 2005).

Whether traditional or problem-based, all of these instructional proposals emphasize the accuracy of mathematical definitions. They all establish the definitions of content by means of logical language. In contrast, previous evaluation of the Mexican context (Ramírez, 1996; 2005) has repeatedly pointed to two comprehension *lacunae* among CS students: (a) translating mathematical language into natural language (and vice versa); and (b) analysing mathematical definitions. Hence, instructional programs of mathematics for CS students should address both topics. Regarding the latter topic in particular, students need to connect different representations of one concept, such as natural language, logical language, mathematical language and pictographic language.

3. Theoretical background: Activity Theory

AT allows mathematics educators to address all the above-mentioned deficiencies and requirements in DM online courses. Currently, there are three generations of AT (Engeström, 2000). The concepts defined by the first generation of AT – mediation, internalization and zone of proximal development (Vygotsky, 1988) – and those proposed by the third generation – learning by expanding, shared zone of proximal development (Engeström, 1987) and situated learning (Lave & Wenger, 1991) – are well established. In contrast, the development and applications of the second generation of AT are less recognised. Our instructional experience specifically draws on the second generation of AT. One of the basic elements of this approach is the accurate definition of the activity structure, through actions and operations (Leontiev, 1984). These concepts enable the study of human activity by facilitating the characterization of the notion of ability, which is a key element in our instructional proposal, both for the design of activities and learning materials, and for the analysis of advancements in learning. In the following subsections, we shall address the instructional decisions step by step, as guided by this theoretical framework.

3.1. The second generation of AT and mathematics teaching in higher education

Leontiev's concept of activity was used by Tallizina (1988), and later by Hernandez (1989) and Valverde (1990), among others, to describe mathematical abilities. For Leontiev, the activity appears as a refinement of the internalization concept and is a constituent element of the psychological subject in both its cognitive (awareness) and its affective and motivational aspects (personality). The activity orients the subject in the objective reality, transforming it into a form of subjectivity. That is, an activity is not just a reaction or a series of reactions; it is a system with structure, development, transitions and internal changes. An activity system produces actions and is, in turn, realized through actions. However, the activity is irreducible to particular actions. Each activity is always linked to a motive (either material or abstract), which responds to a need. The components of human activities are the actions performed by the individuals. The action has an operational aspect (how, by what means can we achieve the objective?), defined by the objective conditions required to achieve the goal of the activity. Activities, actions and operations are dynamic: they can change their 'level' within the macrostructure of the activity under certain conditions.

The design of a learning process departs from the psychological characterization of the activity in terms of its structural components: actions and operations. The educational interpretation of these components is expressed in terms of skills, and requires the mastery of a complex system of actions for self-regulation of the activity. The process of acquiring abilities involves the systematization of the actions they comprise. In turn, this process requires a conscious execution by the subject. The successful execution of the actions indicates the degree of skill development to perform the task. Hence, the subject must master the system of actions in order to fully develop the skill. In other words, we could argue that, for the teaching of mathematical text comprehension, it is essential to characterise the ability and identify the skills it comprises.

3.2. Designing the Orienting Bases of Action

The development of higher mental functions has a social origin (Vygotsky, 1988). This development happens in two separated stages: interpsychological and intrapsychological. Thus, development results from internalized actions. Galperin's Theory of the Stage-by-Stage Formation of Mental Activity (Galperin, 1969) is grounded on Vygotsky's premises and applies them to the instructional context. First, there is the stage of material activity, in which the learner needs to manipulate real objects, and embodied activity, in which the individual can handle models, diagrams and drawings, depending on the learner's age. Second, there is verbalization, when the student needs to repeat the sequence of operations aloud. By rewording, the action moves from the outside to the inside. Finally, the activity can take place at a wholly inner level, which implies thinking.

These evolutionary events can be tailored through the performance of certain guided actions. It is precisely that set of actions which will allow the student and the instructor to monitor and, if necessary, to correct each stage of assimilation. Galperin introduced the term 'orienting basis of an action' (OBA) to refer to the whole set of orienting elements by which the student is guided towards the successful execution of an action (also conceptualized as 'scaffolding' (Samaras & Gismondi, 1998).

In our study, we assume that the ability to read and understand mathematical texts comprises the following actions: (a) the translation of a mathematical statement into natural language, or vice versa; (b) the translation of the statement into the FOL language in order to reveal its structure; and (c) the

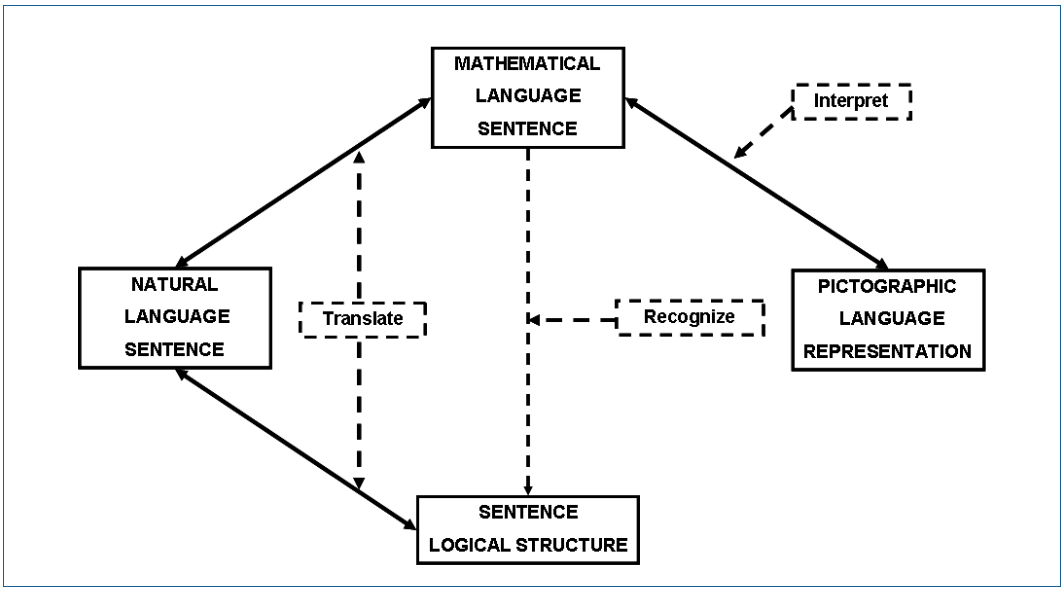

Figure 1: Ability system for the reading of mathematical texts.

representation of the statement in graphical language. To proceed with these steps and correctly interpret the mathematical statements, the students need to master both codes. This identification and characterization of the necessary skills for reading and comprehending mathematical texts provides a basis for designing and implementing online instructional processes (see Figure 1).

4. Addressing the challenges of instructional design

In line with AT, we took three axis elements for the instructional design: course objectives, contents and the scaffolds for the students (OBAs). We shall address each of these elements one by one further below. We shall then explain how these elements were implemented in a Learning Management System (LMS).

4.1. Course objectives

As a result of the problems identified in previous courses, we wanted the students to develop skills to identify and analyze the formal language (logical language and mathematical language itself) and thus represent mathematical concepts and their definitions. This main goal was divided into three objectives:

Students should...

a) Analyze and identify the FOL language in natural language.
b) Identify the mathematical language that is expressed through logical language, and the mathematical entities to which it relates.
c) Interpret definitions in mathematical language, both in formal and pictographic codes.

4.2. Content focus

The basic contents of DM traditional courses in higher education are *Propositional Logic, Predicate Logic, Sets, Relations and Functions*. Logic is usually taught on the basis of a deductive model of content presentation, aiming at demonstration, and using its own rules. In contrast, our course focused on the handling of FOL language and emphasized the process of translation of natural-language statements into logical and mathematical languages. The topic Reading Mathematical Texts was introduced after the Logic unit. For the translation from natural to mathematical language, students received a specific OBA.

The topics *Sets, Relations and Functions* had the following structure: first, the instructor presented a brief reading of the disciplinary area, where target mathematical concepts appeared in their usual contexts. After that, he presented the mathematical subject with standard texts. Thirdly, the students performed the exercise system for each topic, which included two main activities: (a) the analysis of definitions and (b) the use of the corresponding OBAs. Finally, the students received other additional readings of the disciplinary area, in which the target mathematical concepts appeared.

4.3. Orienting Bases of Action

We defined OBAs to help the students in their problem-solving processes. In this course, there were OBAs to translate statements: (a) from natural to propositional-logic language; (b) from natural to predicate language; (c) from natural to mathematical language, and vice versa. Finally, we proposed one OBA for (d) reading mathematical texts and for (e) analyzing the definition of mathematical concepts.

We offered the OBAs throughout the course, along with the material used by the students, introducing them by examples. In the unit Logic, OBAs were characterised and provided in order to develop the translation ability from natural language into FOL language. For the units *Sets, Relations and Functions*, an OBA was provided for the analysis of definitions. The following is an example of a partial implementation of an OBA for the analysis of definitions.

4.3.1. Example of an OBA

Initially, we offered the students step-by-step examples, performing the eight actions of the analysis: (1) differentiating between the expression of the definition in natural language and its expression in mathematical language; (2) identifying the mathematical entities in it; (3) giving examples of objects that both meet and do not meet the definition; (4) finding different ways to represent it; (5) identifying the underlying logical structure; (6) setting its negation; (7) finding the logical equivalence of the definition; and finally, (8) generalizing it.

The process presented to students as a model for using the OBA is outlined in Figures 2 and 3. In these Figures, the actions are listed in the left-hand column and possible responses are illustrated in the right-hand column.

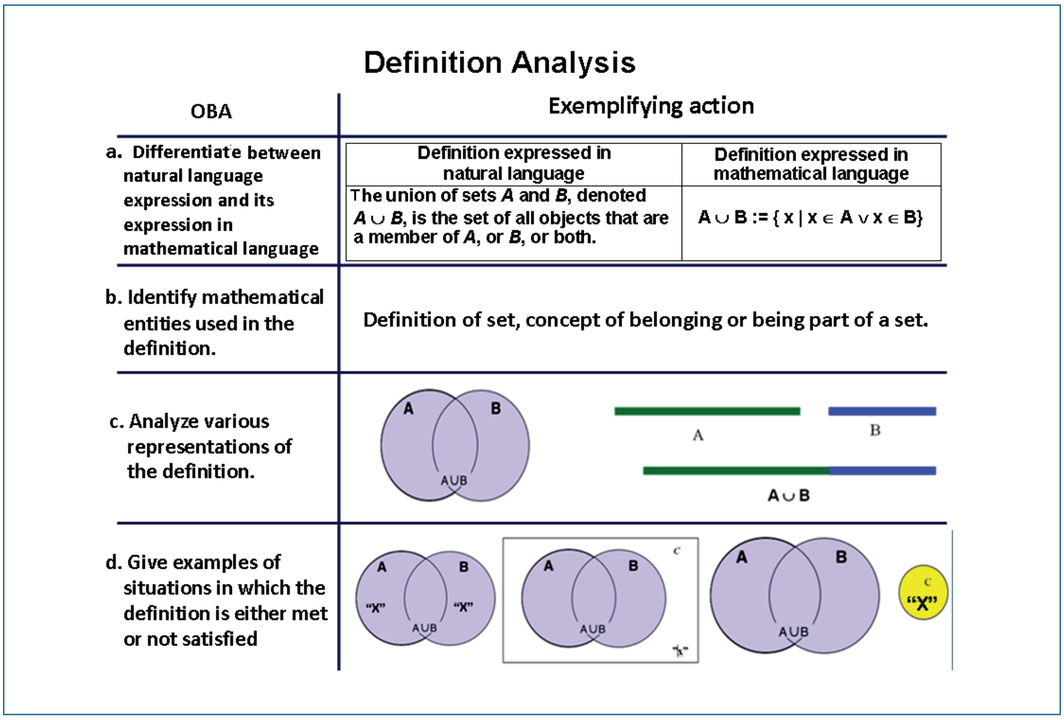

Figure 2: Actions #1 to #4 presented to students as an example for the analysis of a definition.

Definition Analysis

OBA	Exemplifying action
e. Identify the definition's logical structure	If P(x) means that x belongs to A and Q(x) means that x belongs to B, the union definition of the logical structure of two sets is: $P(x) \vee Q(x)$.
f. Set the denial of the definition	From the definition's logical structure $P(x) \vee Q(x)$ we apply the logical negation $\neg(P(x) \vee Q(x))$, getting $\neg P(x) \wedge \neg Q(x)$. So the denial is $\neg(A \cup B)=\{x \mid x \notin A \wedge x \notin B\}$
g. Find the logical equivalences of the definition	In this case an equivalent expression to $P(x) \vee Q(x)$ would be unnatural, we could use $\neg(\neg P(x) \wedge \neg Q(x))$. How can we read this expression in natural language?
h. Generalizing the definition	The union of a finite number of sets is given by "successive unions" $[((A \cup B) \cup C) \cup D) \ldots \cup W]:=\{x \mid x \in A \vee x \in B \vee x \in \ldots \vee x \in W\}$ That can be summarized $\bigcup_{\beta \in \mathrm{B}} A_\beta = \{x \in U \mid \exists \beta \in \mathrm{B} : x \in A_\beta\}$

Figure 3: Actions #5 to #8 presented to students as an example for the analysis of a definition.

This OBA assumes the partial development of the skills to translate statements from mathematical, pictorial and natural languages into each of the three codes. Performing the analysis of definitions provides students with fertile ground for developing the ability to read mathematical texts.

4.4. Techno-pedagogical design of the e-course

In e-learning, many drop-outs are caused by a lack of motivation (Juan, Huertas, Steegmann, Corcoles & Serrat, 2008); therefore, the instructional design of the courses is a key issue in the context of adult education. The term techno-pedagogical design refers to the instructional characteristics of a course as supported by technological devices (Mauri, Colomina & De Gispert, 2009). Indeed, the design of e-learning courses cannot be reduced to the traditional elements of the curriculum, i.e., objectives, content and learning activities and assessment. On the contrary, it must include a reasoned selection and planning of the technological tools that will be used throughout the course, together with a plan for the actual use of these tools and spaces by all participants. Hence, the techno-pedagogical design must include a careful planning of the interactions (instructor-students and among peers) that will be pursued throughout the course.

4.4.1. The LMS

For this course, we used Moodle (V.1.5.8) as the LMS. Moodle presents a flexible structure and leaves many choices open to the course designers and instructors. For example, it is possible to manage different spaces for diverse, flexible groups within the same course. This feature was particularly relevant in this course since it allowed for whole-class group interaction as well as small-group private spaces. The course administrator/instructor takes these decisions, according to the techno-pedagogical design. Furthermore, it allows the management of the course content in individual modules. In this course, we presented each of the five topics separately, in 'week-mode', all of which had the same recursive structure to help the students assume the participation norms.

4.4.2. Interaction design

For the selection and planning of technological tools, it is necessary to determine the interaction among students, and between students and instructor. We adapted the collaborative technique called Team Accelerated Instruction (TAI) (Slavin, 1994) to the Virtual Learning Environment (VLE). In keeping with this technique, the students have to perform three kinds of activities. First, the students must work independently with the learning materials. They are expected to read the course materials and solve the exercises and problems. The second step is for them to work in pairs in order to share and discuss solutions and difficulties with exercises and problems. For this purpose, the students have access to both a synchronous (chat) and an asynchronous (forum) private room on the online platform. The third interaction level covers the whole group. Again, both chat and forum tools support group interaction. The use of each of these spaces and tools is regulated by means of specific participation norms. Figure 4 shows a scheme for the organization of participants and content-learning materials.

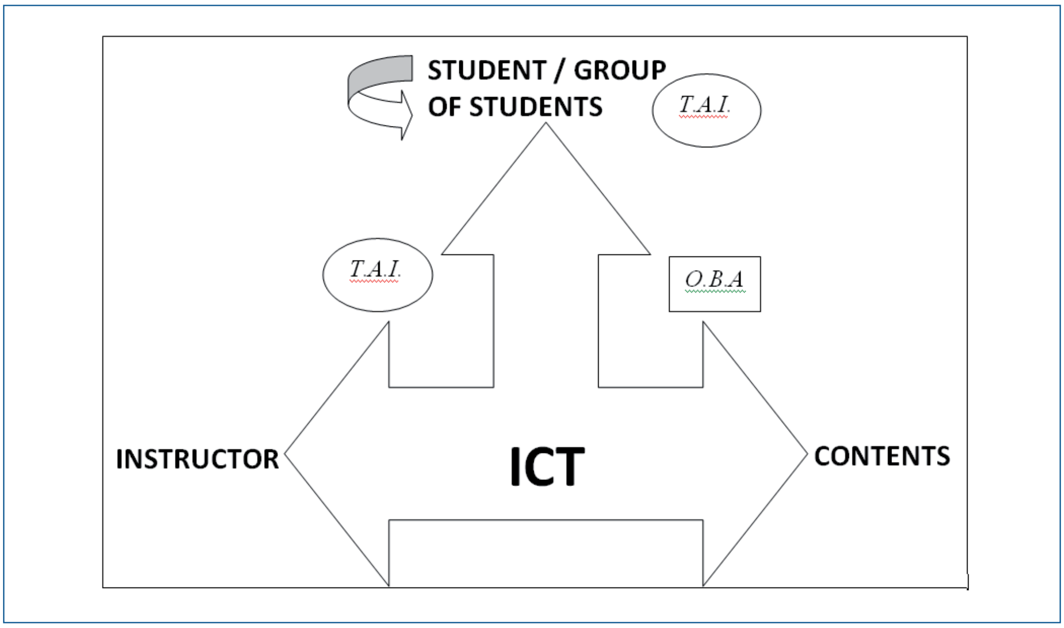

Figure 4: Techno-pedagogical course design.

Student-content interaction. Earlier editions of the course presented a technical difficulty while using chat and forum tools. The participants faced troubles when writing in logical and mathematical language. These problems had been reported previously in similar studies (Smith, Ferguson & Gupta, 2004). Thus, to facilitate mathematical communication, we added an HTML editor with a mathematical equation editor (WIRIS, V.2.1.26) to the chat device (Juárez & Ramírez, 2010). Figures 5 and 6 show the equation editor and some examples of how it can be used.

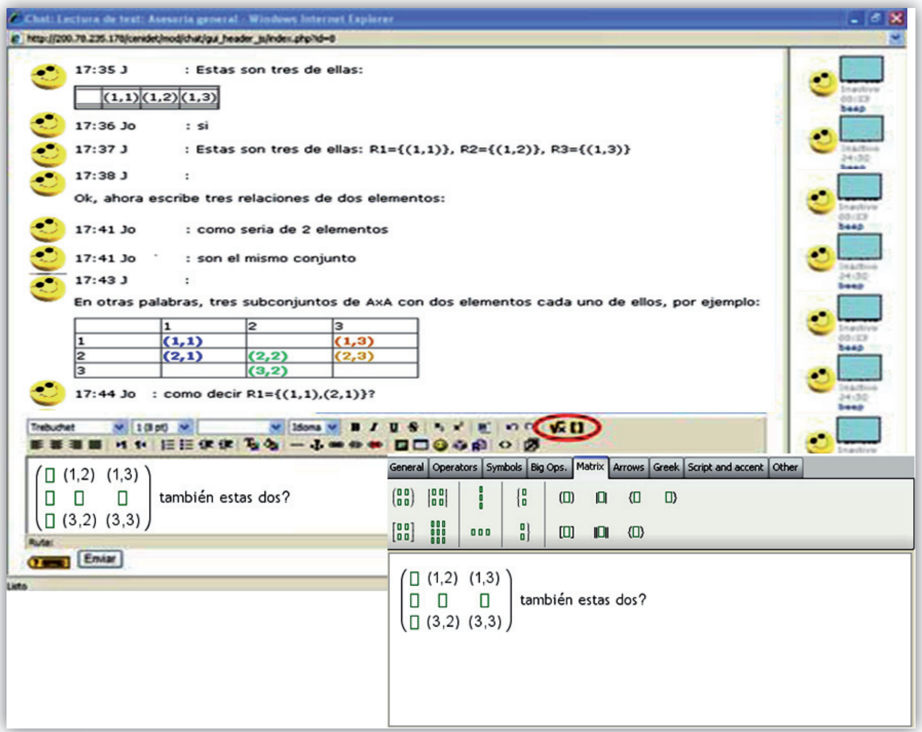

Figure 5: Example of the equation editor and its use in synchronous interaction.

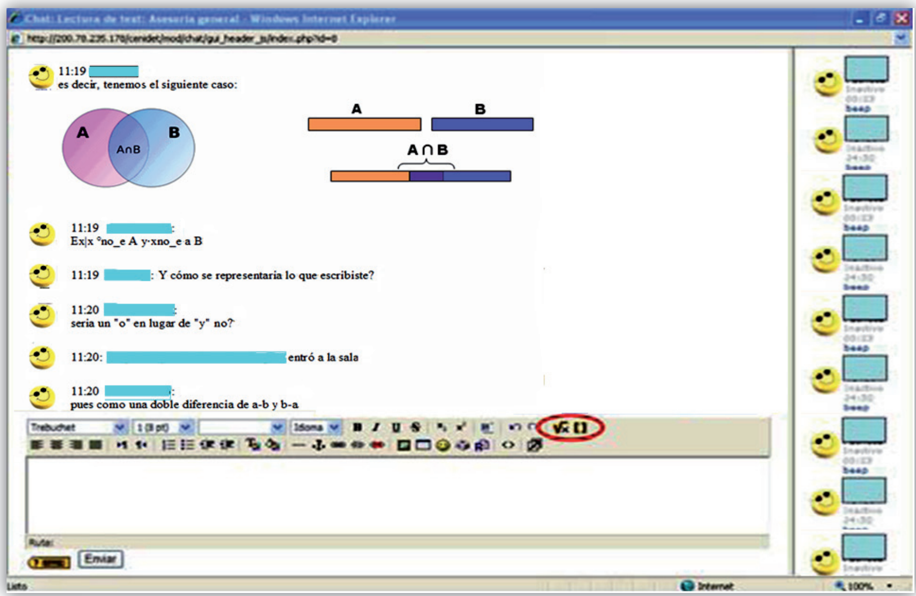

Figure 6: Using WIRIS for online pictorial representation.

Student-instructor interaction. The main area of the course had three communication spaces. First, there was a forum for small-group discussion. This forum provided an asynchronous space to facilitate continuity of discussions and feedback to students by the instructor. Second, there were two chat rooms for synchronous interaction, serving two purposes: there was a first chat room for organized whole-class discussion to resolve doubts under the instructor's guidance, and a second one for technical support.

Student-student interaction. Student-student interaction was designed to occur in pairs and was facilitated by means of different tools. First, there was a chat room for synchronous interaction; second, a wiki for joint resolution of mathematical problems; and third, a database to share results and reflections. Each pair of students could freely decide which tool to use. The group spaces were private to each pair of students; only the instructor could access all the small-group spaces. Hence, he was able to check or participate in the students' interaction, as would be the case in face-to-face TAI situations.

4.4.3. Course structure

The course lasted five weeks, from July to August 2008. A group of 18 students voluntarily accepted to enrol on the master's degree course in CS at CENIDET. The students were CS Engineers from several states across Mexico. The instructor had experience in conventional courses on the subject; in addition, he was familiar with basic technological tools and had participated in the course design.

The course included five units (*Logic and Mathematical Language, Sets, Relations, Functions and Applications*), one per week. The students worked together in pairs, following the TAI model previously presented. If doubts remained after peer interaction, they were brought to the whole-class forum or the whole-class chat.

After each week, the students carried out a self-assessment using a feedback sheet provided by the instructor as a model of resolution. The students had to compare the model with their own resolution in order to identify deviations, strengths and weaknesses. This self-assessment was not graded. The instructor conducted weekly sessions of two hours to offer guidance and clarify doubts. He gave feedback and interacted with the students both synchronously (whole-class chat room) and asynchronously (whole-class forum). Strict turn-taking was established in order to facilitate synchronous interaction in the whole-class chat room. Each student pair had a particular interaction turn with the instructor for 20 minutes. The other participants attended the chat session as observers; they had the chance to 'listen' until the change of turn. This instructional design is presented in more detail in an earlier publication (Remesal, Juárez & Ramírez, 2011).

5. Results: evidence of ability development through the use of Orienting Bases of Action

In order to assess the development of the students' abilities, we performed an interpretive analysis of the following discursive aspects (Lacity & Janson, 1994; Willig, 2004):

1. The students' answers to the exercises.
2. The questions asked in the forum and chat.
3. The comments made in student-student interaction.
4. The results of the students' weekly self-assessment.

We shall now go on to present the specific results of the analysis of data sources #1 to #3, with the purpose of illustrating rather than providing an exhaustive account.

In the following sequence, we can observe an example of a student's development of abilities, that is, of her internalization process (translated from the original language, participants are quoted by pseudonyms). First, we can see how Lois begins the analysis of the definition by indicating what definition she used for inverse functions; then she indicates her doubts regarding its logical structure. She finally tells Mary how she interprets the mathematical definition in natural language. Her explanations and doubts show the acquisition of a skill to organize definitions according to the model shown in the OBA. In this asynchronous interaction, Mary's answer shows how she partially performs the definition analysis. She starts with the definition in natural language and then in mathematical language. Mary provides more elements about what she understands as the logical structure of the definition. She concludes by commenting on the types of properties that a function must satisfy in order to have an inverse function. Despite her analysis, she is unsure about the correctness of the logical structure she proposes:

[Small-group forum. Author *Lois*. 08/02 08:59]

> "Hello Mary,
> What definition in mathematical language did you use for an inverse function? I used the following one, but I'm not sure what its logical structure is:
>
> $$f^{-1}:B[A \mid x = f^{-1}(y),$$
>
> which in natural language is: a function is inverse is any rule or correspondence that allows us to get the values of
>
> $$x \in A \text{ from the values of } y \in B$$
>
> Do you have any other idea?"

[Small-group forum. Author *Mary*. 08/02 11:27]

> "Hello Lois! Good morning,
> Well, let's see if I can help you, OK? Look, my definition of natural language came out like this....
> Be A in B and B in A an inverse function, if and only if the function is bijective, that is, a one to one correspondence and above the co-domain.
>
> $$f \Leftrightarrow f^{-1}$$
>
> in mathematical language...

It's an inverse or invertible function.

$$f = f^{-1} \text{ If,}$$
$$f: A \rightarrow B \text{ and } f^{-1}: B \rightarrow A \text{ if,}$$
$$f: A \rightarrow B\ (\forall x\, \exists y \wedge \forall y\, \exists x)$$

and in its logical structure, I'm not quite sure, but look at what I got…

P(x) ----> Q(x) and Q(x) ----> P(x)

Well, that's what I think, look, because reading the definition, it says that in order for a function to be inverse, you first need to know that it's a function, secondly, check if it has the properties of injective and surjective in order to say that $f: A \rightarrow B$ and $f^{-1}: B \rightarrow A$.

Well, girl, I hope this was helpful,… if it wasn't, tell me and we can talk, I may be wrong and that way we can clear things up, OK?"

Through peer interaction, by sharing doubts and efforts to grasp the OBA in order to handle mathematical content in the multiple communication spaces of the course, the students proved to be progressively internalizing these abilities. In the last units of this course, most students formed definitions according to the OBA through the following actions: first, negating the function; second, translating the function from mathematical language to natural language and vice versa; third, representing the different forms of the function; and finally, representing the logical structure.

[chat room, Mary and Instructor. 07/31]

18:22 *Mary*:	hello teacher, I have doubts regarding the logical structures of each definition.
18:22 *Instructor*:	for example…
18:24 *Mary*:	for example in the surjective function, look, I understood it as follows: one f: A in B is surjective if f (A) is a subset of B and that f(A) is = B
18:25 *Instructor*:	but the basic logical structures are: P and Q; P or Q; P --> Q and the quantifiers, of course
18:30 *Mary*:	it has to be for all "y" (element of the range) corresponds to an x (domain element), then I have to represent its structure using P(x) and Q(x) with the quantifiers".

Figure 7: Example of first steps of OBA use in a chat session.

For instance, the following intervention by Cinthya (Figure 7) shows how she explicitly states the first step for the OBA: "First is the analysis of the definitions. Please, tell me if I'm doing OK." At certain times, the instructor intervened actively to remind the students of the actions that structured the OBAs and guided them to establish a link to the learning content. In the following sequence, for example, the student shows some early recognition of the logical structure of a statement. Thus, the instructor intervenes to remind her of one of the scaffolding actions related to the translation of statements and the recognition of their logical structure.

After the instructor's indications on basic logical structures, Mary remembers the need to use the predicates and quantifiers. Feedback and scaffolding for solving exercises was not only provided by the instructor; at times, other students who participated in chat sessions also contributed to this assistance intervention.

In addition to the chat and forum interaction, the weekly quizzes that the students performed for each unit also informed on the skills development by means of the OBAs, For example, Figure 8 shows part of a student's answer to the first question the third weekly quiz. In this quiz, the student begins rewriting the entire OBA for definition analysis. Then he solves the exercise one step at a time.

Definition Analisys: Reflective Relation

SAGS
Team Lambda

For the definition analysis seven steps should be followed:

a) Distinguish between the definition expressed in natural language and its expression in mathematical language.
b) Think about the mathematical entity or entities referred to in the definition.
c) Analyze various definition´s representations.
d) Give examples of situations that either meet the definition or no meet that definition.
e) Identify the logical structure of the definition.
f) Establish the negation of the definition.
g) Find logical equivalences of the definition.

a) Distinguish between the definition expressed in natural language and their expression in mathematical language.

Natural Language Definition	Mathematical Language Definition
*The relation R is reflexive if every element of set A is related to itself. *A relation **R** on a set **A** is reflexive if any element of **A** relates to himself	Let A be any not empty set. Let R be a relation on A. R is reflexive if and only if $\forall x\ (x \in A \Rightarrow x\ R\ x)$

To complete this step, we must think other ways to say that a Relation is Reflexive and express it in mathematical language.

"R is a reflexive relation on A if and only if all elements of A are related to themselves via R"

RR= is the Set of Reflexive Relations on A.
R = a Relation
$R \in RR \leftrightarrow \forall\ x\ (x \in A \rightarrow x\ R\ x)$

Figure 8: Using the OBA to analyze a definition.

The student's response shows the first step of the definition analysis, expressed in both natural language and mathematical language. The student began his analysis by writing the expressions in both languages in a table. He concluded this step by proposing a different way of expressing the definition in natural language and its corresponding formalization in mathematical language. The student was able to propose his own way of describing the reflexive relationship concept and formalizing it in mathematical language. These actions show how the student used the OBA, and hence, they bear witness to his mastery of the targeted skill to translate a sentence expressed in natural language into mathematical language.

6. Conclusions: assessment of the course design

The need for remedial courses to promote Mexican students' successful participation in the CEDINET master's degree program has been confirmed in recent decades (Ramírez, 1996; 2005). In face-to-face educational situations, the development of mathematical skills is a complex teaching and learning task. Consequently, re-locating it in the virtual context is a risky business in its own right. Particularly, the instructional design and its implementation in an LMS constitute a fundamental challenge to higher education instructors. In this course, the techno-pedagogical design allowed the participants' interaction within the system to be anticipated by promoting student-student synchronous and asynchronous interaction followed by whole-class student-instructor highly structured synchronous interaction. On the one hand, this was pedagogically enabled by the TAI model. On the other, it was technologically facilitated by the flexibility of the LMS and the incorporation of the WIRIS application.

However, and most importantly, the online instructional design presented in this article strongly suggests that the second generation of AT provides useful theoretical elements for promoting the development of the pursued abilities by means of virtual tools. Based on AT, it was possible to define the course objectives in terms of skills, knowledge and application conditions. This in turn allowed the focus to be placed on skills development, using mathematical content as a means, in contrast to traditional approaches of teaching mathematics in higher education, which often focus on content presentation.

In earlier works, we reported the positive assessment that all the participants made of the course (Remesal, 2008). After analyzing the participants' interactions on the virtual platform with regard to undertaking the translation exercises following the OBAs provided, we evaluate the course design positively with respect to three important teaching aspects. First, regarding the content sequence, starting with logical-language proficiency and moving towards an understanding of semi-formalized mathematical texts appeared to be a highly appropriate strategy to facilitate the development of the target abilities. Second, the interaction structure and norms had three positive effects: (1) they allowed the resolution of exercises; (2) they fostered the appropriation of content and the development of abilities; and (3) they encouraged social interaction between pairs of students at a distance. And third, the incorporation of specific software (WIRIS) helped the participants to overcome difficulties in handling algorithmic and pictographic codes in virtual written communication.

Nevertheless, the insufficient duration of the course poses a clear limitation for the full development of the intended abilities, since the development of skills requires gradual practice; indeed, five weeks is too short a time span. In future editions of this course, a longer term (up to eight weeks) should be considered. In addition, we are looking at three possible directions for the next research and instructional steps. First, a longitudinal study is needed to identify how students use the OBA for the analysis of definitions on the Master of CS after the remedial course. This longitudinal project would facilitate the assessment of the remedial course. Second, we intend to expand the remedial course to other related content, such as *Modal Logic* and *Dynamic Logic* of the master's degree program. Finally, audio and videoconferencing tools will be added to upcoming editions in order to establish whether interaction is enhanced through their use.

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