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DOSSIER

Mathematical e-Learning

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Educational technologies are changing the way in which higher education is delivered. These technologies include, but are not limited to, e-learning environments or learning management systems for individual and collaborative learning, Internet resources for teaching and learning, academic materials in electronic format, specific subject-related software, groupware and social networking software. With ubiquitous access to technology and technological innovation, over the last decades not only have we seen the establishment and growth of purely online universities, but we are also now witnessing a transformation in how instruction is being delivered in most traditional face-to-face universities. This transformation is affecting the nature of the courses as well as the degree programs offered by higher-education systems in a global world. These technological innovations have driven the growth of distance-learning opportunities, as students who are time bound – due to job or travel difficulties – or place bound – due to geographic location or physical disabilities – now have the flexibility to access courses and degree programs at their convenience.

e-Learning models are extensively used all over the world. Within mathematics and statistics teaching, educational reforms are widespread both in purely online and in face-to-face education. Many instructors have been encouraged to try new teaching strategies such as online support, interdisciplinary collaborative learning and integration of mathematical and statistical software in their courses. University departments worldwide have been leveraging technological capabilities by creating new engaging curricula that promote conceptual understanding instead of procedural knowledge. Nevertheless, as implementation is not at all easy, especially in mathematics, we are confronted with numerous challenges. Some of these challenges are due to the intrinsic demographic characteristics of so-called 'Internet-generation' students, while others are due to the intrinsic disciplinary nature of mathematics and statistics. In fact, most innovative teaching approaches documented so far have been developed by individuals or by small teams of instructors. These experiences are rarely generalized outside the institution or even maintained over time. Thus, when referring to online mathematics courses, generalization and sustainability of innovative approaches are issues that need to be investigated and promoted by researchers and teaching academics.

In a broad sense, Mathematical e-Learning refers to the use of mathematical software and the Internet to deliver and facilitate instruction of mathematics-related courses. Established technologies (e.g., virtual learning environments and specialized software) enable emerging instructional strategies based on computer-supported collaborative learning. These Web-based strategies are being used in both new and traditional universities to completely teach (either following a synchronous or an asynchronous online mode), partially replace (blended or hybrid learning models) or supplement course offerings in mathematics to a new generation of students. There is little doubt that this new way of teaching mathematics is here to stay and, in fact, its use continues to grow year on year.

With e-learning experiencing what has been characterized as 'explosive growth', there is an urgent need to undertake more research to inform best practices specific to the disciplinary particularities of mathematics e-learning in higher education. While a growing number of publications generically cover e-learning, computer-supported collaborative learning or mathematics education from a more theoretical point of view, few – if any – put emphasis on the practical implementation of mathematical e-learning in higher education. This special issue tries to fill this gap in the literature by identifying and publishing worldwide best practices in the aforementioned field, sharing not only theoretical but also applied pedagogical models and systems. Among others, the goals of the special issue are: (a) to describe experiences on the use of computer-supported collaborative e-learning in mathematical education; (b) to forecast emerging technologies and tendencies regarding mathematical software and its integration into online courses and materials; (c) to explore how learning management systems are contributing to mathematics education online; and (d) to highlight current-edge research in the area.

This *RUSC* special issue contains five articles, selected after a blind peer-review process from almost thirty submitted papers. The selected articles are briefly introduced below:

In "The Role of Digital, Formative Testing in e-Learning for Mathematics: A Case Study in the Netherlands" by D. Tempelaar et al., the authors discuss the importance of formative assessment, in terms of the feedback it provides both to students and instructors of mathematics-related courses, and describe their own experiences while integrating this type of assessment into e-learning platforms.

The article "A Knowledge-Skill-Competencies e-Learning Model in Mathematics" by G. Albano addresses the emergent issue of how to successfully model mathematics-related competencies in an e-learning environment. The author presents a model, based on knowledge and skills representations, which defines a personalized learning experience to promote students' competencies in mathematics.

In "Activity Theory and e-Course Design: An Experience in Discrete Mathematics for Computer Science", J. L. Ramírez et al. present an interesting e-learning experience involving a higher-education course on mathematics. The course design is based on two theoretical approaches: while the content-related design is supported by different concepts of Activity Theory, interaction between participants is designed following Slavin's Team Accelerated Instruction model.

The article "Distance Training of Mathematics Teachers: The *EarlyStatistics* Experience" by M. Meletiou-Mavrotheris and A. Serradó analyzes how information and communication tools could be employed to improve the quality and efficiency of teacher training in statistics education. The authors also point out lessons learned from their own experience with *EarlyStatistics*, an online course in statistics education which was offered to European elementary and middle school teachers.

In "On How Moodle Quizzes Can Contribute to the Formative e-Assessment of First-Year Engineering Students in Mathematics Courses", M. Blanco and M. Ginovart describe their experience with the use of Moodle's quiz module, and discuss the utility of this tool for the formative assessment of students.

This special issue also contains a review, written by H. Cuypers, of the book *Teaching Mathematics Online: Emergent Technologies and Methodologies*, which has recently been published by IGI Global.

Finally, we would like to thank the authors and reviewers of this special issue for their collaboration and prompt responses to our enquiries, which enabled completion of this manuscript in a timely manner. We gratefully acknowledge the editor at *RUSC*, Ms Elsa Corominas, for her help and encouragement during the entire editing process of this *RUSC* special issue.

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Dossier "Mathematical e-learning"

ARTICLE

The Role of Digital, Formative Testing in e-Learning for Mathematics: A Case Study in the Netherlands

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Abstract

Repeated formative, diagnostic assessment lies at the heart of student-centred learning, providing students with a continuous stream of information on the mastery of different topics and making suggestions to optimize the choice of subsequent learning activities. When integrated into a system of e-learning, formative assessment can make that steering information instantaneous, which is a crucial aspect for feedback in student-centred learning. This empirical study of the role of formative assessment in mathematics e-learning focuses on the important merit of integrating these assessments into a system of state or national testing. Such tests provide individual students with crucial feedback for their personal learning, teachers with information for instructional planning, and curriculum designers with information on the strengths and weaknesses in the mastery states of students in the program and the need to accommodate any shortcomings. Lastly, they provide information on the quality of education at state or national level and a means to monitor its development over time. We shall provide examples of these merits based on data from the national project ONBETWIST, part of the Dutch e-learning program Testing and Test-Driven Learning.

Keywords

interim assessment, bridging education, mathematics, heterogeneous international education, mathematics program reforms

El papel de los exámenes formativos digitales en el aprendizaje virtual de matemáticas: un estudio de caso en los Países Bajos

Resumen

La repetida evaluación diagnóstica y formativa es uno de los elementos clave del aprendizaje centrado en el alumno, ya que ofrece a los estudiantes un flujo continuo de información sobre su nivel de conocimientos en distintas materias y permite optimizar la posterior elección de actividades de aprendizaje. Cuando se integra en un sistema de aprendizaje virtual, la evaluación formativa puede convertir esta información en instantánea, lo que constituye un aspecto crucial para el retorno de información en un aprendizaje centrado en el alumno. Este estudio empírico sobre el papel de la evaluación formativa en el aprendizaje virtual de matemáticas se centra en la ventaja de integrar estas evaluaciones en un sistema nacional o estatal de exámenes. Estos exámenes proporcionan a los estudiantes una información crucial para su aprendizaje personal; suministran a los profesores los datos necesarios para llevar a cabo la planificación docente; y ofrecen a los encargados de elaborar los planes de estudio la información necesaria sobre las fortalezas y las debilidades de los estudiantes de cada programa y la necesidad de solucionar cualquier deficiencia. En último lugar, ofrecen información sobre la calidad de la enseñanza a escala nacional o estatal y son un *medio para controlar su desarrollo a través del tiempo. Daremos ejemplos de todas estas ventajas según los datos del proyecto nacional ONBETWIST, que forma parte del programa de aprendizaje virtual holandés «Los exámenes y el aprendizaje basado en exámenes».*

Palabras clave
evaluación intermedia, cursos puente, matemáticas, educación internacional heterogénea, reforma de los programas de matemáticas

Introduction

According to a recent, domain overarching meta-analysis of empirical educational studies (Hattie, 2008), feedback is the most effective instructional mechanism. Feedback can have many different sources, and in student-centred learning, students' mastery or lack of mastery to perform a specific task is an important part of that feedback. Formative assessment is a means to repeatedly assess a student's mastery in order to establish the subsequent learning step, and its importance is extensively documented, in the context of both traditional learning (Donovan et al., 2005; Pellegrino, et al., 2001) and e-learning (Juan et al., 2011). Recently, there has been some interest in systematically combining formative assessment with the use of state or national tests. In the U.S., this is termed 'interim assessment' (Beatty, 2010). According to the U.S. National Research Council, interim assessments "are assessments that measure students' knowledge of the same broad curricular goals that are measured in annual large-scale assessments, but they are given more frequently and are designed to give teachers more data on student performance to use for instructional planning. Interim assessments are often explicitly designed to mimic the format and coverage of state tests and may be used not only to guide instruction, but also to predict student performance on state assessments, to provide data on a program or approach, or to provide diagnostic information about a particular student. Researchers stress the distinction between interim assessments and formative assessments, however, because the latter are typically embedded in instructional activities and may not even be recognizable as assessments by students …" (Beatty, 2010, p. 6).

Continuous evaluation processes are at least as crucial in mathematics education as they are in other disciplines (Donovan et al., 2005; Taylor, 2008; Trenholm et al., 2011). Beyond assessment for development and assessment for achievement, both of which are generally recognized as important assessment functions, formative assessment in mathematics education functions as 'transition' or 'placement' assessment, particularly in the first year of university education (Taylor, 2008). In their comparative study of long-term online mathematics teaching experiences, Trenholm et al. (2011) provide four major case studies, all of which point to continuous assessment as a key factor of success. However, empirical studies into the effect of formative assessment in mathematics education remain scarce (Wang et al., 2006).

In the Netherlands, SURF, the Dutch collaborative organisation for higher education institutions and research institutes aimed at innovations in ICT, initiated the nationwide program Testing and

Test-Driven Learning to stimulate the design and use of such interim assessments, among other things. Part of this program is the ONBETWIST project (http://www.onbetwist.org/), focusing on mathematics learning, both in the transition from high school to university, and in the first year of university, using e-learning with the support of these interim assessments. The ONBETWIST project builds on earlier projects, such as SURF projects NKBW (http://www.nkbw.nl/) and TELMME (www.telmme.tue.nl), and EU projects S.T.E.P. (www.transitionalstep.eu/) and MathBridge (http://www.math-bridge.org/). All these projects focus primarily on the design and use of mathematics e-learning tools to facilitate the transfer from high school to university, e.g., for international students who have been educated in school systems whose premises differ considerably from those on which the university curriculum is built. Offering flexible bridging courses in mathematics when the inflow of students is too heterogeneous in terms of prior mathematics mastery to start immediately with class-based regular university teaching is, in short, the main aim of all these initiatives. Reviews of some of these endeavours can be found in Brants et al. (2009), Rienties et al. (2011) and Tempelaar et al. (2008). In our companion paper, Tempelaar et al. (2011), we report on the outcomes of bridging education in the context of the NKBW project for one Dutch university. This university is a typical exponent of European internationalisation of higher education, where international students account for more than 70% of the total. Although most of these students are not very international in terms of the geographical distance they have to bridge, there is huge diversity with respect to the high school education they have received. Secondary school systems, even in neighbouring countries like the Netherlands, Germany and Belgium, are very different, producing major heterogeneity in mathematical knowledge and skills that prospective students have. Such heterogeneity means that there is a considerable need for bridging education in the transfer from secondary to university education, and it offers an outstanding case to demonstrate the advantages of interim assessment. While the companion paper focuses on the remedial education component, designed as a voluntary mathematics summer course, this paper investigates the use of digital, formative tests for diagnostic aims in the same population of international students. The empirical context of this study refers to the use of entry tests generated within the national NKBW and ONBETWIST projects (full versions of the tests can be found in the open-access ONBETWIST question database, available at http://moodle.onbetwist.org/), where subjects for the empirical study are selected from one university, which is characterized by a strong international orientation and large year classes.

The aim of this study is to add to the limited body of empirical studies into the effects of formative assessment in mathematics education, thereby focusing on its role in the first year of university education, where assessment, beyond development and achievement functions, plays an additional and important transition or placement role.

The UM mathematics summer course

Since bridging education takes place before students participate in the interim assessments, a short introduction to the summer course is required in order to understand its impact on performance in the assessments. The voluntary mathematics summer course is constructed around the test-steered, adaptive, e-tutorial: ALEKS (Assessment and LEarning in Knowledge Spaces) College Algebra module. The tool makes use of server-based computing and can be characterised as supporting individual distance learning. The ALEKS system (see also Doignon & Falmagne, 1999; Falmange et al., 2004; Tempelaar et al., 2006) combines adaptive, diagnostic testing with an e-learning and practice tutorial in several domains relevant to higher education. In addition, it provides lecturers with an instructor module, where students' progress can be monitored in both learning and assessment modes.

The ALEKS assessment module starts with an entry assessment in order to evaluate a student's knowledge of the domain. Following this assessment, ALEKS delivers a graphic report analyzing the student's knowledge within all curricular areas of the course. The report also recommends concepts on which the student can begin working; by clicking on any of these concepts or items, the student gains immediate access to the learning module. See Figure 1 for a sample of the learning report.

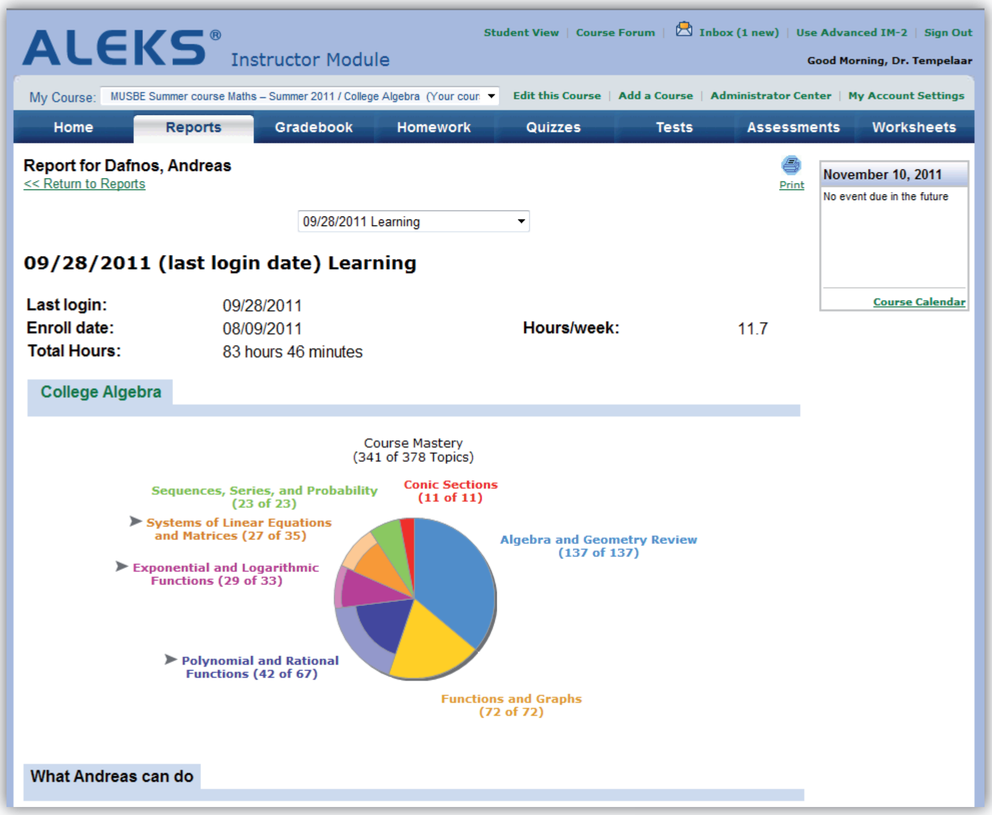

Figure 1. Partial sample of an ALEKS learning report

Some key features of the assessment module are that all problems require the student to produce authentic input, all problems are algorithmically generated, and assessment questions are generated from a carefully designed repertoire of items, thus ensuring comprehensive coverage of the domain. The assessment is adaptive: the choice of each new question is based on the aggregate of responses to all previous questions. As a result, the student's knowledge state can be found by asking only a small subset of the possible questions (typically 15-25). Both the principles of the UM summer course, and the use of the e-tutorial ALEKS, are described in more detail in Tempelaar et al. (2011). An important aspect for this study is that the summer course is extra-curricular; offered before the regular program starts, participation can only be voluntary. As a consequence, three different groups of students can be distinguished: those not participating in the summer course (*NoSC*), those successfully participating in the summer course (*SCPass*), and those enrolling the summer course but not reaching a sufficient level of achievement (*SCFail*). To distinguish between passing and failing summer course participants, a mastery level of 55% of the lessons contained in the ALEKS module was used.

Participants

This study is based on the investigation of five cohorts, of about equal size, of first-year students at a Business and Economics School in the south of the Netherlands (academic years 07/08, 08/09, 09/10, 10/11 and 11/12). Programs offered by this school deviate from mainstream European university education in two important ways: the student-centred learning approach of problem-based learning and a strong international orientation (the programs are offered in the English language and mainly attract international students). Of the 3,900 students in these five cohorts, 71% have an international background (mostly European, and just over 50% from German-speaking European countries) and 29% are Dutch. Of these students, 36.7% are female and 63.3% are male. The mean age of the students was 20.12 years, with a range of 17-31 years, though most students were in their teens: the median age was 19.82 years. They were all enrolled on a business and economics program.

A large majority of these students took part in the administration of at least one diagnostic entry test: 3,014. A small minority of the students took part in the voluntary summer course: 622, of which 267 passed and 335 failed (did not achieve a 55% mastery level in ALEKS).

After finishing the summer course in late August, the regular program of bachelor's degree studies in International Business and International Economics started in early September. Both programs begin with two eight-week (half semester) integrated, problem-based learning designed courses, each having a 50% study load. The first course is an introduction to organizational theory and marketing. The second course, called Quantitative Methods I or QM1, is an introduction to mathematics and statistics. The very first activity in the QM1 course is to administer the mathematics entry test. The coverage of the QM1 course mirrors the circumstance that strong heterogeneity in mathematics mastery, due to students being educated in different national systems and at different mathematics levels, necessitates a fair amount of repetition. Most of the topics covered are repeats of those taught in grades 11 and 12 of Dutch secondary schooling, basic mathematics level (the last two years of high school), with some time devoted to new topics. There is no overlap between QM1 and the content of the summer course, since that content covers those topics taught in grades 7-10 of secondary schooling (middle school and first year of high school).

The major component of heterogeneity in mathematics mastery is caused by the level of mathematics schooling in high school. European countries generally distinguish between two different levels of high school mathematics: basic and advanced. Of the students in this study, 28.1% did their high school education under the Dutch national system, called VWO (pre-university education), and were taught mathematics at one of two different basic levels (A1 or A1,2) or one of two advanced levels (B1 or B1,2). The lowest level, A1, prepares students for studies in arts and humanities, but does not qualify them to take social sciences studies such as business or economics, so what remains is only the higher basic level: DutchA12 (18.6%). Another two tracks are at advanced levels: DutchB1 (4.5%, preparing for life sciences studies) and DutchB12 (2.3%, preparing for technical studies). Due to a reform in mathematics education in the Netherlands, students taking the advanced track in high school from the last two cohorts (10/11 and 11/12) were educated in an undifferentiated advanced track: DutchB (5.4%). A majority of students (53.1%) was educated in a German-speaking high school system. That system again has two different levels of prior mathematics education, the advanced level or *Leistungskurs*, and the basic level or *Grundkurs*. Students taking the basic track have a further choice to select mathematics as one of their four subjects in the final examination or *Abitur* (students in the advanced track always do so). As a consequence, there is one advanced track: GermanLK (13.9%), and two basic tracks: GermanGKA (25.0%) and GermanGKnA (13.8%), where the last category has opted out with regard to final examination. Again, in the last two cohorts, a new but very small category of students can be distinguished owing to a reform in mathematics education in some of the German states: the merger of basic and advanced tracks into one single, undifferentiated level of mathematics education: GermanUndif (0.8%). In comparison to other European universities, there is a rather large share of students having an International Baccalaureate (IB) diploma (6.9%). IB again allows one advanced level (HL) to be distinguished from two basic levels (SL and StudiesSL), generating the categories IBMathHL (1.5%), IBMathSL (5.1%) and IBMathSSL (0.3%, but excluded from this study due to its small size). The remaining students (11.9%) are educated within a national system outside the Dutch or German-speaking part of Europe. For this last category, students were asked to classify their own prior mathematics education at the level of either mathematics major or mathematics minor. This results in the categories OthMathMajor (6.2%) and OthMathMinor (5.7%).

Interim assessments

In this study, we investigate the role of two different interim assessments. Both are designed for use in the transfer from high school to university and, for that reason, are labelled as entry assessments in the two projects for which they were designed. We shall adhere to that convention.

The first entry test is called the NKBW entry test, designed within the SURF NKBW project. In that project, secondary education and tertiary education representatives cooperated on the design of these entry tests, giving the entry tests the unique characteristic of being based on a shared opinion of what prospective students should master when graduating from high school and entering university. That is, the NKBW tests are both entry and exit tests. Tests were developed for different tracks of high school mathematics education; since mastery of mathematics at higher basic track level is required, we applied that type of entry test. That 16-item test comprises four topics: algebraic skills (*AlgebraicSkills*), logarithms and exponentials (*Log&power*), equations (*Equations*), and differentiation (*Differentiation*). In this study, we focus mostly on the topic of algebraic skills, since deficiencies in the mastery of these skills appear to have a great impact on study success in the first year of university, and the topic is beyond the scope of most forms of refreshment education provided at the start of regular university education in many programs. Such refreshment topics typically include more advanced topics from senior high school, whereas algebraic skills are taught in junior high school, if not in primary education. Algebraic skills constitute a main part of the summer course program. NKBW entry tests have been available since 2009, and the two cohorts 09/10 and 10/11 of students investigated here have participated in this test.

The other entry test applied is the one designed by the TELMME project partners: the three Dutch technical universities. For that reason, the entry test is called the 3TU test. Since the test is based on the mastery of the advanced track of high school mathematics education, items belonging to the topic differentiation and integration were deleted from the test. The remaining categories are algebraic skills, logarithms and exponentials, and equations, and total 14 items. Written for a more advanced target audience, items have a somewhat higher level of difficulty than items in the NKBW entry test. They also have a stronger focus on skills mastery, whereas the conceptual understanding of mathematics is somewhat more prominent in the NKBW test. The 3TU test was administered in all five cohorts of UM freshmen, and thus provides a better basis for analysing developments over time.

Results

Prior education and the 3TU and NKBW entry tests

Figure 2 demonstrates the development over the years of diagnostic entry test scores in the topic *AlgebraicSkills* of the main prior mathematics education groups in our study. We shall focus on the component algebraic skills in most of this section, since it is at the heart of the project. However, the analysis of total scores in the entry test results in rather similar outcomes, with identical patterns, but at a slightly lower level.

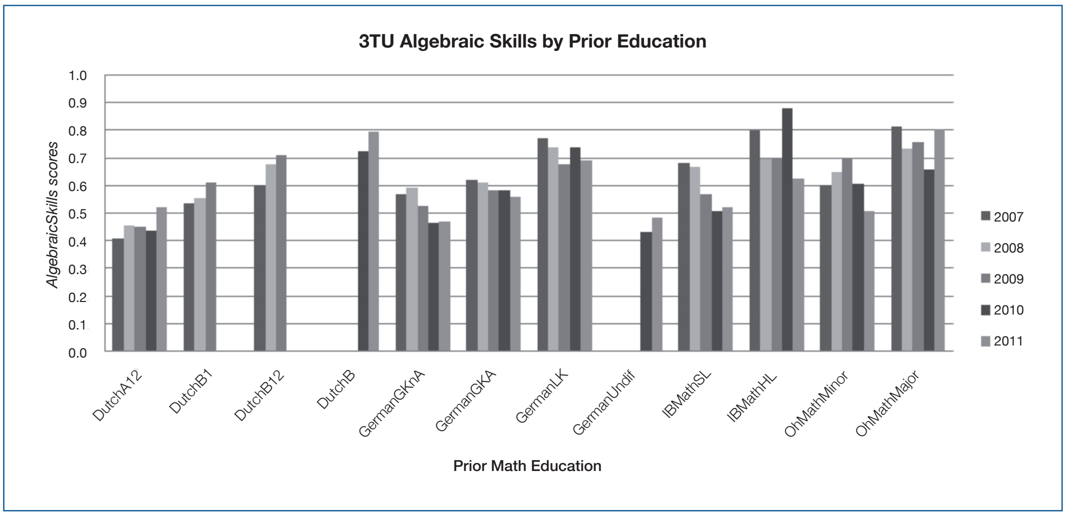

Figure 2. *AlgebraicSkills* scores from the 3TU entry test, by prior mathematics education

When the entry tests were administered for the very first time in 2007, we were surprised to find such a major underperformance of national (Dutch) students compared to international students. For example, national students with the most advanced prior mathematics education, DutchB12, scored no more than 60%, against a 62% score for German students educated at basic level (GermanGKA) and a 77% score for German students educated at advanced level (GermanLK). Needless to say, the scores of Dutch students from the less advanced tracks were even lower: 41% for DutchA12 and 53% for DutchB1. Indeed, they were the lowest of all other types of prior education. However, given the raison d'être of our national bridging project, this outcome was not that surprising. In fact, it did provide justification for the project, since Dutch secondary education proved to lack in preparing students for university, especially in the area of algebraic skills, not only in an absolute sense, but also in a relative sense, when comparing Dutch students to international students.

Since 2007, several remarkable developments have occurred. School reforms in Dutch secondary mathematics education have improved the performance of advanced track students year after year, for both the B1 and B12 tracks. The merger of both of these tracks into one DutchB track was another successful step in terms of mastery of algebraic skills: students from that broad track achieved 72% and 79% scores, higher than ever before. And by doing so, they approached the score of German advanced track students (74%, GermanLK). Scores of Dutch basic track students, however, remained at the very lowest level.

Amongst the three different types of international prior education, radically different developments can be observed. Mastery levels of the advanced tracks are relatively stable and high (greater variability present in the scores of IBMathHL, though that may simply be due to sampling variability, given the smaller size of this group, 15 on average). The OthMathMajor category seems to demonstrate decreasing scores, but, being a residual category, this is not easy to interpret. Mastery levels among basic track students do, however, signal a decline over the years for both German and IB students, with very marked developments for the GermanGKnA and IBMathSL groups. As a consequence, mastery levels among all tracks of basic mathematics education (except OthMathMinor) converge to worryingly low levels – ranging between 40% and 50% – that have been present in the Netherlands for some time. In contrast to the success of the Dutch educational reform, the reform in Germany of removing different tracks to create an undifferentiated system seems to be less successful: the score is certainly not higher – and is more likely to be lower – than that of the basic track still in existence in other states. However, this group is somewhat small to place a lot of trust in this outcome.

The assessment of the German educational reform also depends on the type of entry test applied: changing to the NKBW entry test, which is based rather more on conceptual understanding and somewhat less purely on skills, German undifferentiated system students score midway between the basic and advanced tracks (60% versus 59% and 69%). In addition, the other educational reform is assessed differently: the new DutchB group scores similarly to, or even slightly lower than, students from the advanced track the year before. Besides being more conceptually oriented, the NKBW entry test is clearly less difficult (scores are uniformly higher) and less discriminative between the basic and advanced tracks than the 3TU entry test: see Figure 3.

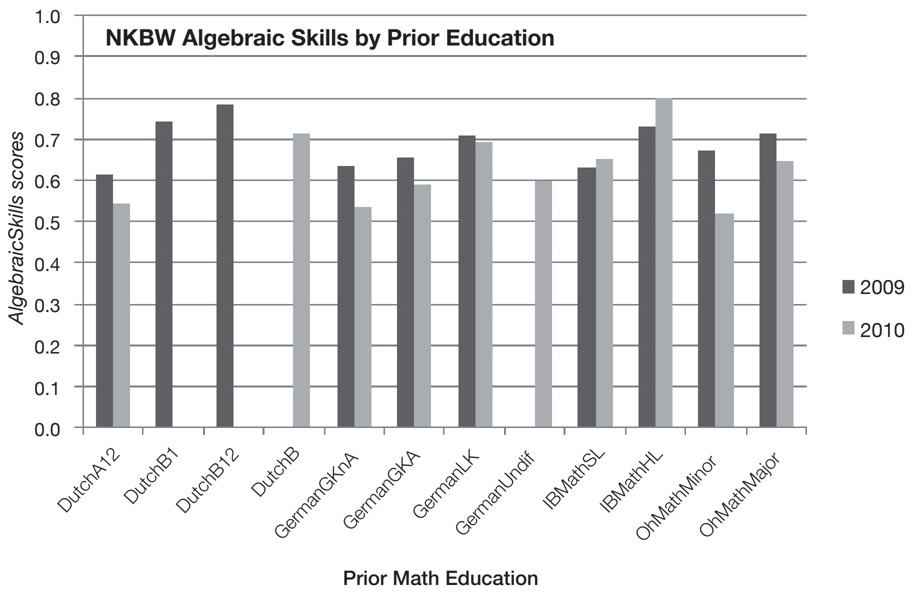

Figure 3. *AlgebraicSkills* cores from the NKBW entry test, by prior mathematics education

How large the differences between international mathematics educations can be with regard to the mastery of very basic algebraic skills is illustrated by the scores of two entry test items from this category in the 3TU entry test. The items themselves are provided too: see Figures 4 and 5.

AlgebraicSkillsNo2: $\dfrac{x^2 - x}{x^2 - 2x + 1}$ equals: a. $\dfrac{x}{1-x}$ b. $\dfrac{1}{2x-1}$ c. $\dfrac{-x}{-2x+1}$ d. $\dfrac{x}{x-1}$

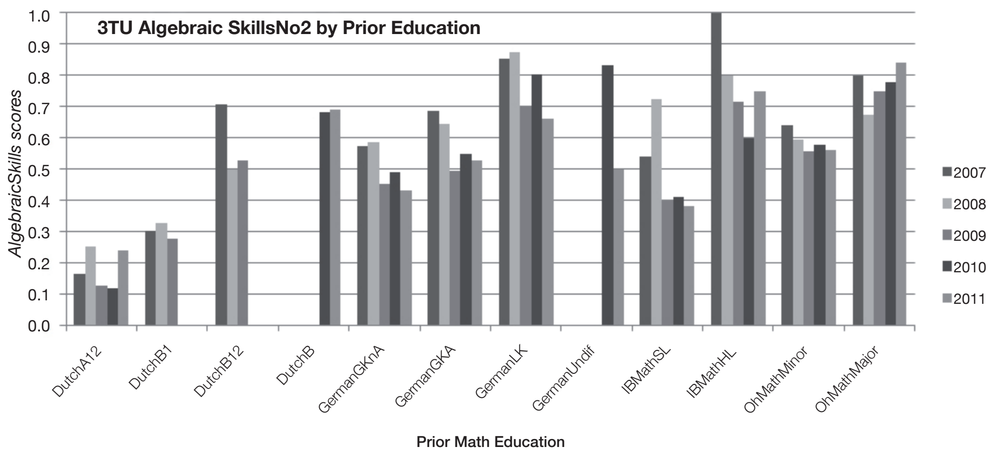

Figure 4. *AlgebraicSkillsNo2* scores from the 3TU entry test, by prior mathematics education

AlgebraicSkillsNo3: $\dfrac{x}{x+1} + \dfrac{x}{x-1}$ equals: a. $\dfrac{2x}{2x-2}$ b. $\dfrac{2x^2}{x^2-1}$ c. $\dfrac{2x^2}{1-x^2}$ d. $\dfrac{2x}{x^2-1}$

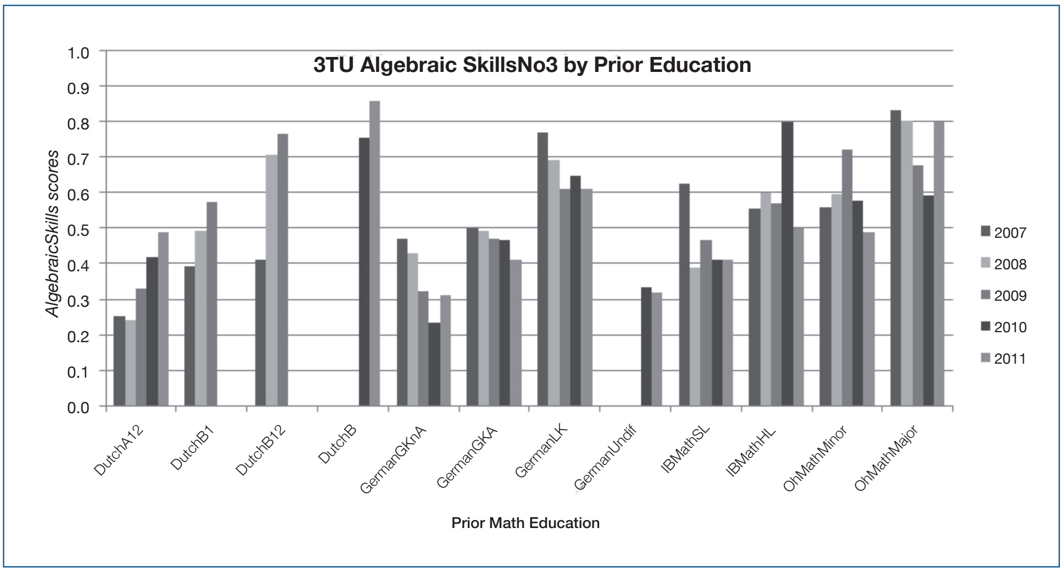

Figure 5. *AlgebraicSkillsNo3* scores from the 3TU entry test, by prior mathematics education

Where the *AlgebraicSkillsNo3* scores are not beyond guessing level in some group and year combinations, we at least observe improved mastery over time, especially for the Dutch students, who were the weakest in 2007. In contrast, scores for *AlgebraicSkillsNo2* in the Dutch basic track are even lower than guessing rate, and do not indicate any sign of improvement over time: students continue to become strongly attracted to the third answer option, apparently following the strategy of eliminating equal quadratic terms in numerator and denominator. Beyond very strong differences between outcomes for the Dutch and other European educational systems, both items, but especially the first one, also demonstrate considerable mastery differences between basic and advanced track students. This is remarkable in its own, since algebraic skills are typically taught in junior high school, to students in both the advanced and basic tracks.

Summer course participation and the 3TU and NKBW entry tests

Since the mathematics bridging course offered by the program runs in the summer, participation is voluntary, which allows student performance in the entry tests to be compared for three different groups of students: successful summer course participants, unsuccessful summer course participants and non-participants. Figure 6 and 7 contain student performance scores in both types of entry test in the *AlgebraicSkills* section and, as reference material, in two other topics in the NKBW entry test. Figure 6 makes clear that there is a strong effect of successful participation in the summer course. The true effect is even greater than the figure suggests, since students educated at the basic level are overrepresented in the group of summer course participants, whereas students educated at the advanced level are overrepresented in the group of non-participants (according to the aim of the summer course). Part of this overrepresentation is visible in the scores of unsuccessful summer course participants: in three of five cohorts, their mastery level is significantly lower than the mastery level of the non-participants, indicating that these students initially made the right decision to register for the bridging course, but failed to materialize that decision.

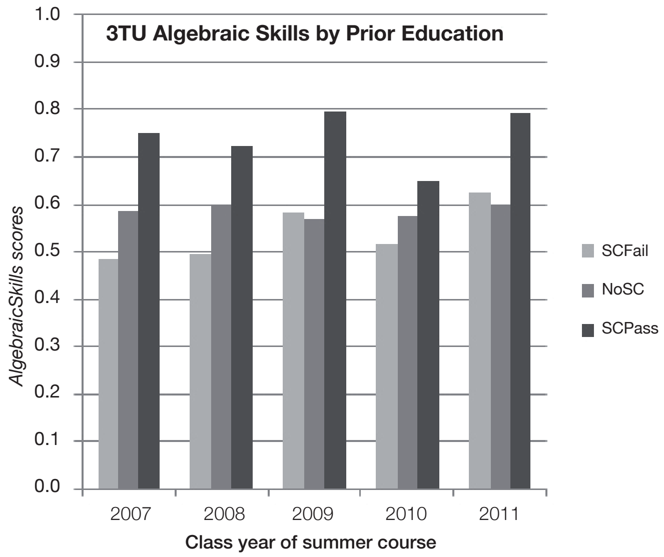

Figure 6. *AlgebraicSkills* mastery in the 3TU entry test, by summer course participation

The first panel of Figure 7 confirms that impression, be it that effects are much weaker when measured with the NKBW entry test *AlgebraicSkills* section. The second panel indicates that items in the *Logs&Powers* section contain a stronger effect of bridging education. And the third panel is added to check the adequacy of comparisons of this type. The third panel contains the items of the *Differentiation* section, not part of the summer course. For comparisons between the three groups to be valid, no effect of bridging education is to be expected in this third panel, which is indeed the case.

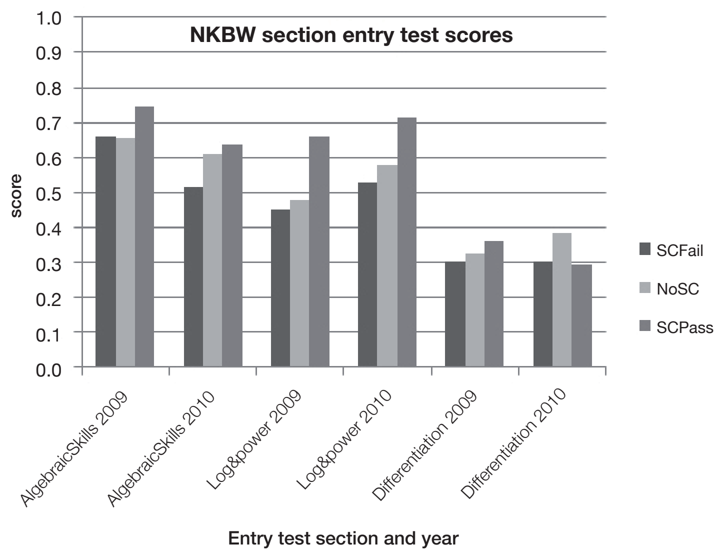

Figure 7. *AlgebraicSkills, Logs&Powers & Differentiation mastery* in the NKBW entry test, by summer course participation

Does participation in the summer course help students achieve academic success, beyond getting higher scores in purely formative tests like the two types of entry test? The answer is clearly affirmative, as visible in Figures 8 and 9. Figure 8 contains the scores in the final exam for both sections of that exam: mathematics and statistics (maximum score being 20). The effects of successful summer course participation are substantial, in both sections, where the true effects are again expected to be stronger than the visible effects, given that weaker students are overrepresented in the summer course. Differences in final mathematics exam scores between successful summer course participants and non-participants are statistically significant at 1% level in all class years, except for 2008 and 2011. Differences in final statistics exam scores are statistically significant at 1% level in class years 2007 and 2009.

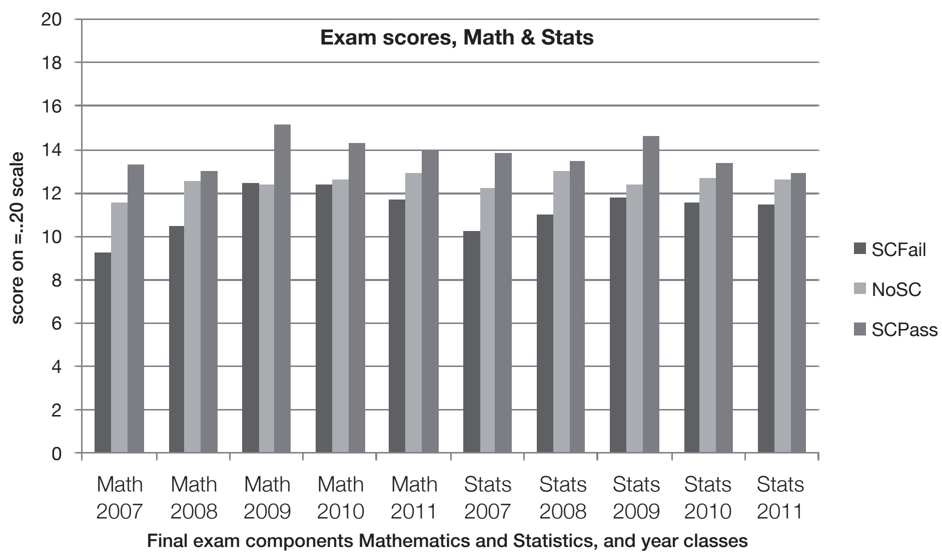

Figure 8. Score in final mathematics exam and final statistics exam, by summer course participation

The strongest effects are visible in Figure 9, containing the passing rates for the QM course. Since most students score in the region of 55% (required to pass), the effects of summer course participation are stronger on passing rates than on absolute score. Differences in passing rates between successful summer course participants and non-participants are statistically significant at 1% level in all class years except for 2008, where significance is at the 10% level.

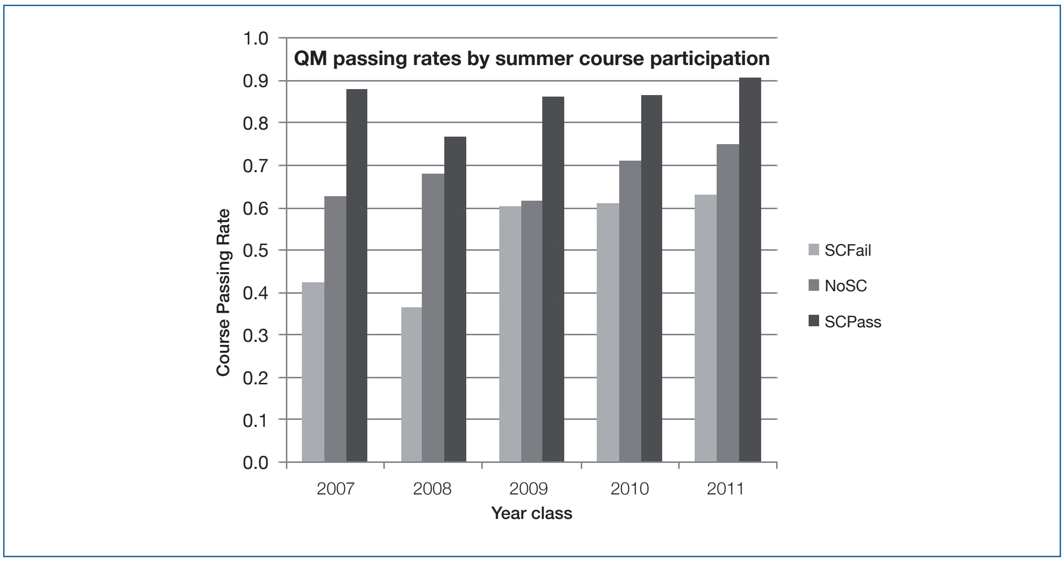

Figure 9. QM passing rate, by summer course participation

Prior education and summer course participation, and the 3TU entry test

In order to properly disentangle the combined effects of prior mathematics education and summer course participation, it is necessary to analyse the effect of bridging education separately for students of each of the different types of prior education. Figure 10 provides the outcomes of one sample of such an analysis.

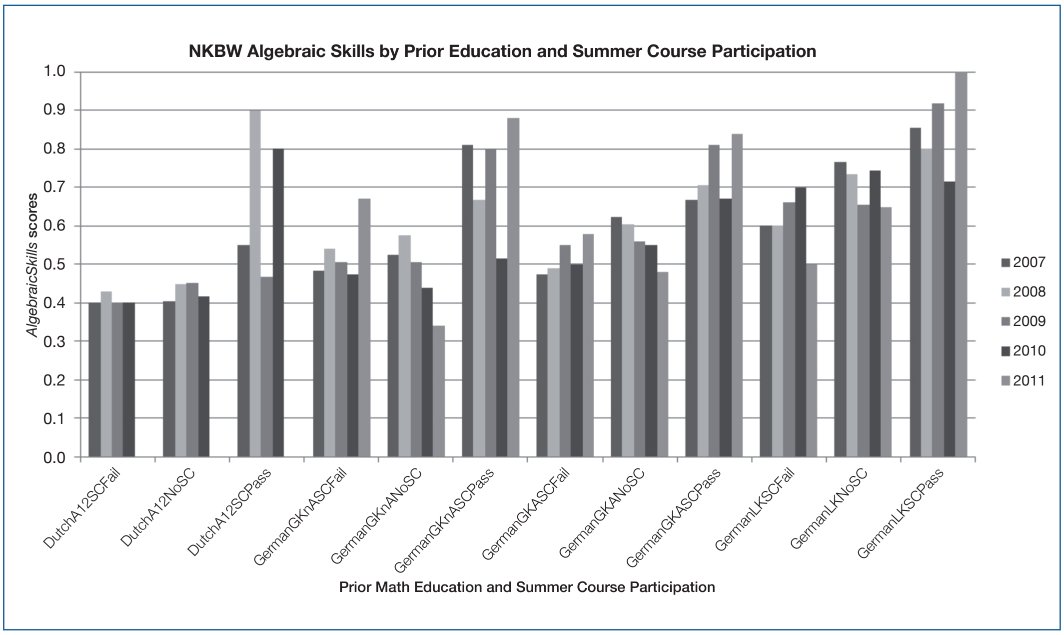

Figure 10. *AlgebraicSkills* scores in the 3TU entry test, by prior education and summer course participation

Since only a minority of students participated in bridging education, comparison is restricted to those prior education categories that had a sufficient number of students (five) in each of the three categories *NoSC*, *SCFail* and *SCPass*. Prior education categories that satisfy that constraint are DutchA12, GermanGKnA, GermanGKA and GermanLK. With the exception of the last category, these are all categories of basic tracks of mathematics education. German students are overrepresented, partly because many German students interrupt their studies after finishing high school and go to university only after a break of often two or more years. These students, even if educated at advanced level, regard the summer course as an effective refresher of their mathematics mastery. For all four prior education categories, Figure 10 contains three panels, corresponding to failing, non-participation and passing the summer course. As expected, we observe that entry test scores demonstrate both a prior education effect and a summer course participation effect. The summer course effect seems to be weakest among advanced track students, which is no surprise: beyond some refreshment, these students cannot gain much from participating in the bridging course. Stronger effects are to be found for students educated at the basic level. But beyond the systematic differences, there is more sampling variability present, due to smaller samples, which makes interpretations from these decomposed data less easy.

Cluster analysis of 3TU entry test scores

A very different approach to analysing data derived from entry test takes is to look at groups of students with similar score patterns for the different items in the test. We did this by applying cluster analysis; Figure 11 contains a graphical representation of the outcomes of such a cluster analysis.

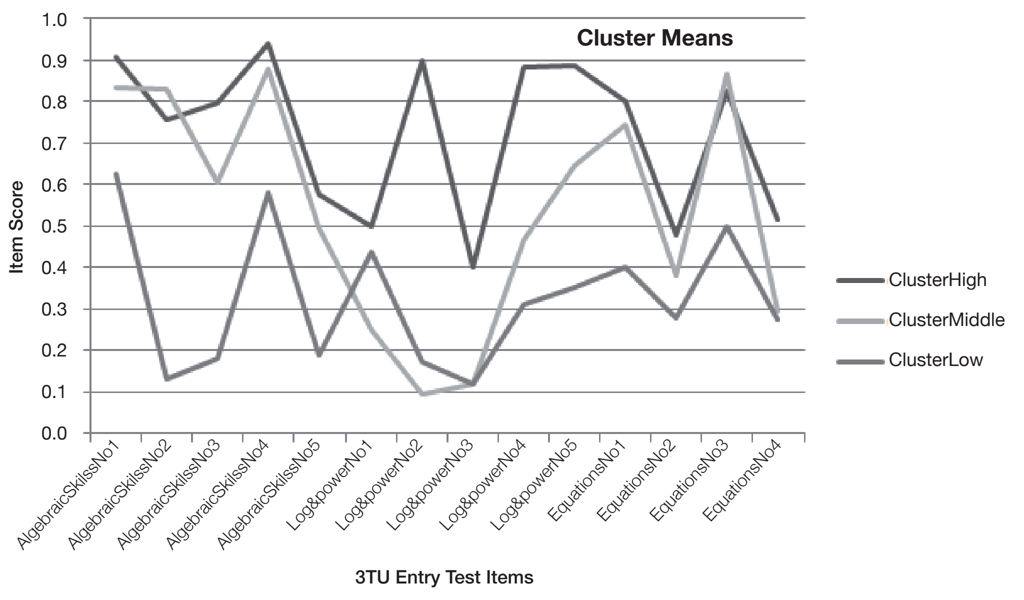

Figure 11. Clustering of students in high, middle and low clusters based on five years of 3TU test takes

The analysis is performed on all test takes together, by adding all five cohorts. In the analysis, each student is allocated to one of three clusters, where the clusters are calculated to maximize variation between clusters and to minimize variation within clusters. Such cluster analysis can be repeated within each of the prior education groups; in this contribution, we shall limit ourselves to the outcomes of cluster analysis applied to all groups together. In most of these cluster analyses, distinguishing three clusters works quite well and, in most cases, these clusters are easily interpretable. As one can see from Figure 11, the clusters represent high scoring students, low scoring students, and a group of students whose performance is between the two. The latter middle group is by far the most interesting one, especially since the students perform similarly to the high scoring students for some items, and similarly to the low scoring students for other items. In Figure 11, students in the middle cluster score similarly to those of the high cluster for items belonging to the *AlgebraicSkills* section, with the third item (discussed earlier) as a potential exception. In contrast, students in the middle cluster score similarly to students in the low cluster, or even lower, in items in the *Log&power* section. They score highly again in the *Equations* section, especially on the third item, which requires them to find the zeros of a standard quadratic equation. Deviant patterns are here for the second question, which acts as a kind of trick questions: it asks for the number of different zeros of a third order polynomial, in which two zeros coincide. And the last question, where beyond solving an equation, students need to know how to find a tangent line. In short, middle cluster students act on the same level as high cluster students when items can be solved by the straightforward application of regular solution strategies taught in high school, but fall back to the level of low cluster students when items deviate from the regular pattern of class exercises.

Conclusions and discussion

The repeated use of formative, diagnostic tests is a crucial component of any mathematics e-learning program, providing the feedback required for the optimal steering of individual learning. The use of broad 'interim' assessments for these purposes brings many additional advantages. First, it allows the strengths and weaknesses of different prior education tracks to be distinguished for programs attracting large numbers of international students educated in very different high school programs. Second, when the heterogeneous inflow is accommodated by implementing bridging education, it allows the effects of prior education and of remedial education to be properly disentangled. Finally, it allows different clusters of students with very different patterns of mathematics mastery to be distinguished. Doing so provides important information – beyond that for the individual students – for instructional planning, for regular curriculum design, for the implementation of bridging programs, for the streaming of education and even for admission regulations. Inferential statistical analyses indicate that first-year students using these formative assessments and participating in the summer course (based on such a formative assessment strategy) are substantially more successful (in the sense of statistical significance) than students who do not.

Both students and instructors evaluate the facilities of online formative assessment as highly positive. However, it is difficult to assess the evaluation of the developmental and placement functions of formative assessment separately from the evaluation of the achievement functions. As in many other programs, online formative assessment is introduced together with online low-stakes testing in the form of quizzes. Positive evaluation of formative assessment is therefore not to separate from the appreciation of low-stakes testing and the availability of online tools to prepare these quizzes.

Future research will have a dual focus. First, formative tests, specifically entry tests, provide crucial feedback with regard to the mathematics proficiency of students from different backgrounds. Given the recent major reform in Dutch secondary mathematics education, longitudinal monitoring of the mathematics proficiency of prospective students from different secondary education systems will continue to serve an important function. Second, future research will focus on the role formative tests can play in both providing continuous and instantaneous feedback to students, and in making education itself more adaptive, with the aim – in both instances – of optimising the learning process.

Acknowledgments

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References

BEATTY, A. (Rapporteur) (2010). *Best Practices for State Assessment Systems Part I: Summary of a Workshop*. Washington, D. C.: National Academy Press.

BRANTS, L.; STRUYVEN, K. (2009). "Literature review on online remedial education: A European perspective". *Industry and Higher Education*. Vol. 23, No 4, 269-276.

DOIGNON, J. P.; FALMAGNE J. C. (1999). *Knowledge spaces*. Berlin: Springer.

DONOVAN, M. S.; BRANSFORD, J. D. (eds) (2005). *How Students Learn: Mathematics in the Classroom*. Washington, D. C.: National Academy Press.

FALMANGE, J.; COSYN, E.; DOIGNON, J.; THIÉRY, N. (2004). *The assessment of knowledge, in theory and in practice*. [Accessed: 1 December 2010]. <http://www.aleks.com/about_aleks/Science_Behind_ALEKS.pdf>

HATTIE, J. (2008). *Visible Learning. A synthesis of over 800 meta-analyses relating to achievement*. London: Routledge.

JUAN, A. [et al.]. (2011). "Teaching Mathematics Online in the European Area of Higher Education: An instructors' point of view". *International Journal of Mathematical Education in Science and Technology*. Vol. 42, No 2, pages 141-153.

PELLEGRINO, W.; CHUDOWSKY, N.; GLASER, R. (eds) (2001). *Knowing What Students Know: The Science and Design of Educational Assessment*. Washington, D. C.: National Academy Press.

RIENTIES, B. [et al.] (2011). "Describing the current transitional educational practices in Europe" [submitted article]. *Interactive Learning Environments*.

TAYLOR, J. A. (2008). "Assessment in First year university: A model to manage transition". *Journal of University Teaching & Learning Practice*. Vol. 5, No 1.

TEMPELAAR, D. T.; RIENTIES, B. (2008). "Remediating summer classes and diagnostic entry assessment in mathematics to ease the transition from high school to university". In: *Proceedings of Student Mobility and ICT: Can E-LEARNING overcome barriers of Life-Long Learning.* Maastricht: FEBA ERD Press. Pages 9-17.

TEMPELAAR, D. T. [et al.] (2006). "An online summer course for prospective international students to remediate deficiencies in math prior knowledge: The case of ALEKS". In M. Seppällä; O. Xambo; O. Caprotti (eds). *Proceedings of WebALT2006*. Technical University of Eindhoven: Oy WebALT Inc. Pages 23-36.
<http://webalt.math.helsinki.fi/webalt2006/content/e31/e157/e161/6_zDR2j2uQcB.pdf>

TEMPELAAR, D. T. [et al.] (2011). "Mathematics bridging education using an online, adaptive e-tutorial: preparing international students for higher education". In: A. A. Juan; M. A. Huertas; S. Trenholm; C. Steegmann (eds). *Teaching Mathematics Online: Emergent Technologies and Methodologies*. IGI Global.

TRENHOLM, S. [et al.] (2011). "Long-Term Experiences in Mathematics E-Learning in Europe and the USA". In: A. A. Juan; M. A. Huertas; S. Trenholm; C. Steegmann (eds), *Teaching Mathematics Online: Emergent Technologies and Methodologies*. Hershey, PA: IGI Global. Pages 236-257.

WANG, K. H.; WANG, T. H.; WANG, W. L.; HUANG, S. C. (2006). "Learning styles and formative assessment strategy: enhancing student achievement in Web-based learning". *Journal of Computer Assisted Learning*. Vol. 22, No 3, pages 207-217.

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UOC Universitat Oberta de Catalunya

RUSC Revista de Universidad y Sociedad del Conocimiento
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Dossier "Mathematical e-learning"

ARTICLE

A Knowledge-Skill-Competencies e-Learning Model in Mathematics

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Abstract

This paper concerns modelling competence in mathematics in an e-learning environment. Competence is something complex, which goes beyond the cognitive level, and involves meta-cognitive and non-cognitive factors. It requires students to master knowledge and skills and at least some measurable abilities, which Niss calls 'competencies'. We present a model that exploits the innovative technological features of the IWT platform to define a personalised learning experience allowing students to increase their competence in mathematics. It is based on knowledge and skills representations by means of a graph metaphor, and on a theoretical framework for modelling competence.

Keywords

mathematics learning, knowledge, skill, competence, competency, e-learning

Conocimientos, destrezas y competencias: un modelo para aprender matemáticas en un entorno virtual

Resumen

Este trabajo se centra en la competencia matemática en un entorno de aprendizaje virtual. La competencia es algo complejo, que trasciende el nivel cognitivo, e implica factores metacognitivos y no cognitivos. Exige de los estudiantes su dominio sobre conocimientos y destrezas y, por lo menos, sobre algunas capacidades medibles, a las que Niss llama «competencias específicas» [competencies]. El modelo que presentamos utiliza las innovadoras características tecnológicas de la plataforma IWT para definir una experiencia de aprendizaje personalizado que permite a los estudiantes aumentar su competencia en matemáticas. Se basa en la representación de conocimientos y destrezas mediante metáforas gráficas y en un marco teórico para modelar la competencia.

Palabras clave

aprendizaje de matemáticas, conocimientos, destrezas, competencia, competencia específica, aprendizaje virtual

1. Introduction

Competence in mathematics is something complex, hard to define, which requires students to master not only knowledge and skills, but at least some measurable abilities, which Niss calls 'competencies' (detailed in section 2). In this paper we face the problem of mathematics teaching-learning in an e-learning environment, with particular respect to competencies. The author has extensive experience in undergraduate blended courses supported by the e-learning platform IWT (Intelligent Web Teacher), which allows personalised Units of Learning (UoLs) for each learner to be created and delivered by means of an explicit knowledge representation (see section 3). The latter has been improved in order to make a clear distinction between knowledge and skills (Albano, 2011a). Competency modelling requires a different approach and further work, since it is based on knowledge and skills and goes beyond the cognitive and meta-cognitive levels of both. In this paper, starting from the assumption that the learning of competencies comes from the engagement of learners in suitable Learning Activities (LAs), we propose a model that is able to generate and update suitable *templates* associated with the learning of a certain competency. Moreover, we give a complete framework of how the knowledge-skill-competency model should work in the IWT context. In particular, the IWT learning personalisation features can be exploited to personalise the delivery of LAs, so as to engage learners in those activities that best match their individual cognitive states and learning preferences.

The paper is organised as follows: sections 2 and 3 give an overview of the theoretical and technological frameworks, respectively; section 4 describes a model for knowledge and skills in mathematics learning, based on a multi-level graph representation of the domain; section 5 describes a model for competencies, framed in Dubinsky research on undergraduate mathematics education (RUME); section 6 shows how the three models work and integrate; section 7 analyses costs and benefits; section 8 explores the opportunities for future research; and section 9 draws some conclusions.

2. Theoretical framework

Many authors (Weinert, 2001; D'Amore, 2000; Godino, J.; Niss, 2003) have tried to explain competence in mathematics. According to Niss (2003), "possessing mathematical competence means having knowledge of, understanding, doing and using mathematics". All these authors agree that it is not something to be taught; rather, it is a long-term goal for the teaching-learning process. It is something complex and dynamic, which requires mathematics domain knowledge of a declarative-propositional type and of a procedural type, that is, knowledge (to know) and skill (to know how), but at the same time goes beyond cognitive factors. Table 1 shows a list of some basic requirements for the distinction between knowledge and skills:

Table 1. Classification of the main types of mathematical content.

Content type	*Knowledge*	*Skill*
Definition	Statement	Procedure/computation
Theorem Theorem	Statement Proof	Procedure/computation Procedure/computation
Algorithm Example (counter)	 Description	Performance of the algorithm
Exercise Problems		Computational skills Solving standard problems

In order to make the notion of mathematical competence more factual, we can consider a mathematical competency as a clearly recognizable distinct major constituent in mathematical competence (Niss, 2003). Niss has distinguished eight characteristic cognitive mathematical competencies, adopted by PISA 2009 (OECD, 2009). They correspond to relational mathematics (Skemp, 1976), which consists of reasoning, thinking, problems, and processes. This is reflected by 'relational comprehension', which means to know why. The following table lists them in two clusters (Niss, 2003):

Table 2. Clusters related to cognitive mathematical competencies.

The ability to ask and answer questions in and with mathematics	*The ability to deal with mathematical language and tools*
Mathematical thinking competency	Representation competency
Problem handling competency	Symbols and formalism competency
Modelling competency	Communication competency
Reasoning competency	Tools and aids competency

3. Technological framework

From a technological viewpoint, we refer to the IWT platform that we used in our practices. It is a software platform for distance learning, equipped with Learning Content Management System (LCMS) and adaptive learning system features. It allows teaching-learning experiences to be both personalized and collaborative by means of the explicit representation of knowledge and the use of Web 2.0 techniques and tools. Realized at the Italian Pole of Excellence on Learning & Knowledge and marketed by MOMA[1], this platform, which is not open source, has been adopted by various Italian universities and high schools.

3.1 The main features of IWT

Personalisation of the learning process is made possible in IWT by means of three models: Knowledge, Learner, Didactic.

The Knowledge Model (KM) is able to intelligibly represent the computer and the information associated with the available didactic material. It makes use of:

1) Ontologies, which allow the formalization of cognitive domains through the definition of concepts and relations between the concepts. They consist of graphs, whose nodes are the concepts of the cognitive domain and whose edges represent the relations HasPart, IsRequiredBy and SuggestedOrder, designed by domain experts using a specific editor available in IWT (Figure 1).

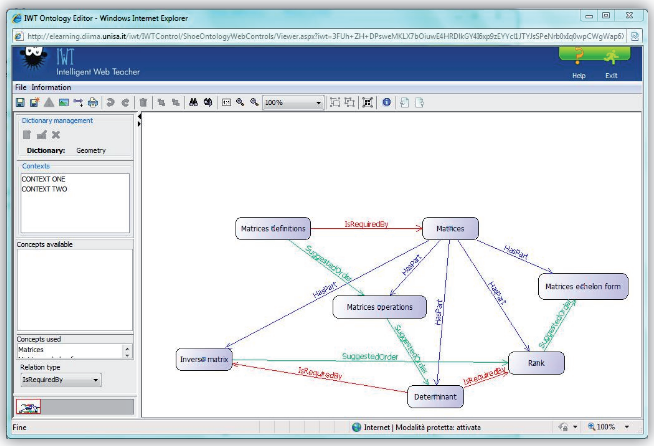

Figure 1. An example of an ontology for the topic Matrices.

2) Learning Objects (LOs), consisting of "any digital resource that can be reused to support learning" (Wiley, 2000).

3) Metadata, which are descriptive information tagged to each LO in order to associate it with one or more concepts in an ontology (Figure 2, red box). Further information refers to educational

1. http://www.momanet.it/index.php?lang=en

parameters such as LO typology (video, text, slide, etc.), context (high school, college, training, etc.), type of interaction (expositive, active, mixed) and its level, difficulty and semantic density.

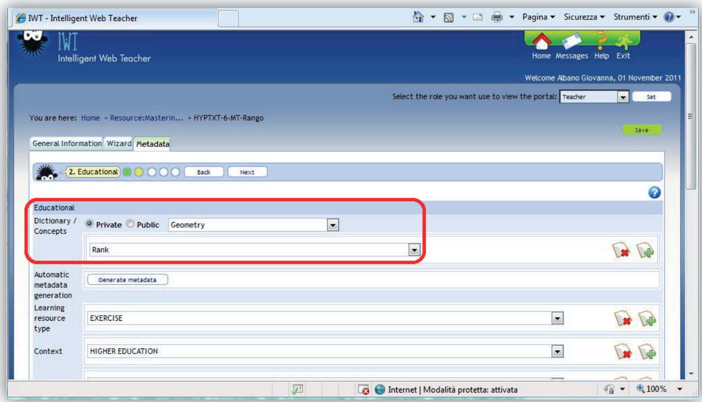

Figure 2. An example of metadata (zoom on Ontology/Concept association).

The Learner Model (LM) allows a user profile to be managed (Figure 3). The user profile automatically captures, stores and updates information on individual learners' preferences and needs (e.g., media, level of interactivity, level of difficulty, etc.) and their cognitive states (that is, concepts of a knowledge domain that have already been learnt).

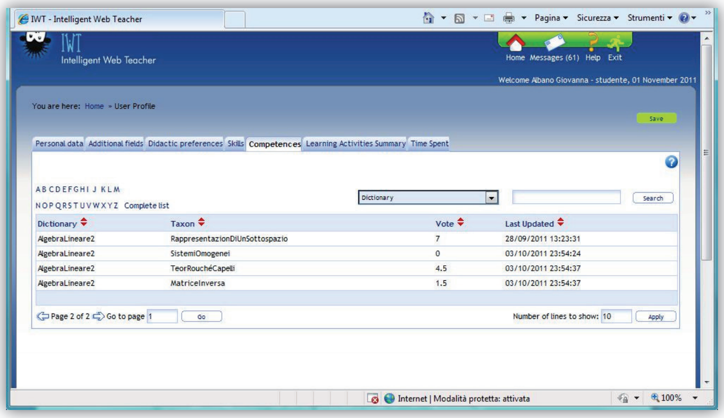

Figure 3. An example of a learner profile.

The Didactic Model (DM) refers to a pedagogical approach to learning (e.g., inductive, deductive, learning by doing, etc.). Currently, it is associated with specific LO typologies (for instance, a simulation refers to inductive didactic learning) and it is stored both in the LO metadata and in user profiles (as preferred LO typologies).

3.2. How IWT works

IWT allows both guided and self-regulated learning. The former consists of standard courses (e.g., Geometry, Calculus) and the latter allows learners to express their learning needs in natural language (e.g., learning to solve linear systems). In both cases, IWT gives rise to tailored UoLs, taking advantage of the previous models (Albano, 2011b; Albano et al., 2007; Gaeta et al., 2009). Domain experts (i.e., teachers) first define a number of suitable specifications for courses or learning needs, choosing or editing a suitable ontology for the course topics. They then set suitable learning goals (e.g., one or more target concepts on the chosen ontology) and finally choose certain parameters for the teaching flow (e.g., pre-test, how many intermediate tests, educational context). The UoL is generated at run-time from IWT, when a student accesses it for the first time, through the following steps: the ontology is used to create the list of concepts needed to reach the target concepts of the course, then user profile information allows this list to be updated according to the cognitive state, and the LOs to be chosen. These LOs are those whose metadata best match the learner preferences. Further, the UoL is dynamically updated according to the outcomes of intermediate tests.

4. Multi-level graphs to model knowledge and skills learning

The current use of ontology in IWT corresponds to a rough version of teaching according to 'fundamental nodes'. With this term we refer to "those fundamental concepts which occur in various places of a discipline and then have structural and knowledge procreative value" (Arzarello et al., 2002). In mathematics education, teaching by fundamental nodes means "to weave a conceptual map, strategic and logic, fine and smart" (e.g., Figure 4), where each concept is the goal of a complex meshed system, where no concept stands completely alone and each of them is part of a relational web rather than a single "conceptual object" (D'Amore, 2000).

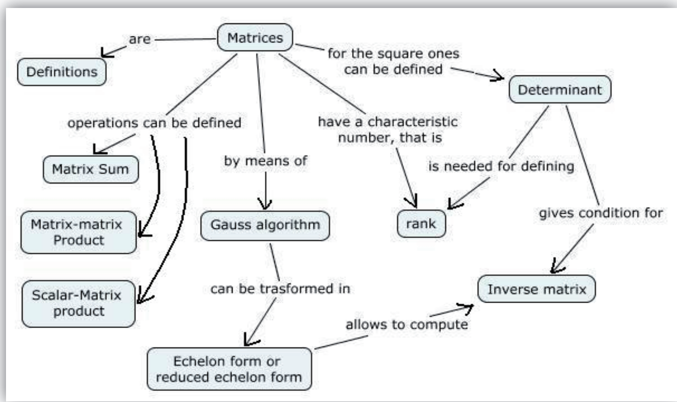

Figure 4. An example of a conceptual map for the topic Matrices.

As we can see, the distinction between the knowledge and the skill levels is made clear by the relations between the nodes (i.e., the edges). Edges in IWT ontologies cannot do the same. So the two levels are flattened onto the nodes, thus associating LOs for both of them.

In order to overcome this restraint, we propose the use of a multi-level graph representation (Albano, 2011a). At the first level, the fundamental nodes are seen as 'roots' of a further two graphs (ontologies), where the levels of knowledge and skill are made explicit.

- Knowledge level (Figure 5), where the nodes correspond to definitions, theorems, examples, etc. (Table 1), and the possible relations, mandatory (continuous lines) or not (discontinuous lines).

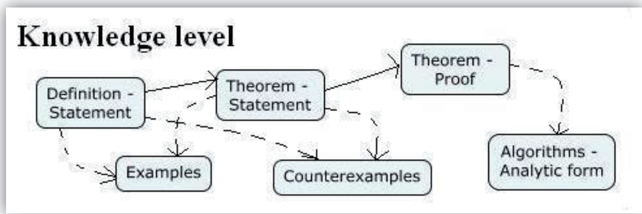

Figure 5. Generic ontology at the knowledge level.

- Skill level (Figure 6): where the nodes correspond basically to computational methods and standard problem-solving capabilities (Table 1).

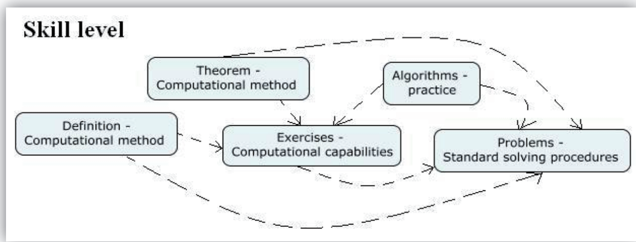

Figure 6. Generic ontology at the skill level.

Moreover, a third level related to competencies can be devised:

- Competency level: where the nodes correspond to those competencies for the fundamental node 'root' (Table 2).

In the following section, we shall take the competency level and its modelling into consideration.

5. Dubinsky's cycle to model competency learning

According to the theoretical framework, we assume that competencies develop from students' engagement in LAs. This is why, in order to model competencies, we refer to Dubinsky's RUME framework (Asiala et al., 1996). It consists of a cycle of three interrelated elements, which are theoretical analysis, instructional treatment and data collection/analysis.

Let us see what they mean in our context. Starting from a concept, we can single out one or more associated competencies. Then, we can implement a LA aimed at getting students to practice them. Thus, we can start the cycle described below:

Theoretical analysis

The theoretical analysis aims to propose a competency learning model, that is, a description of mental construction processes used by learners in their understanding of the competency, called Genetic Decomposition (GD). Such GD is strictly dependent on the content to which the competency is applied (e.g., representation competency has a different meaning if it concerns a set of real numbers or the lines in a 2D space) and it is not necessarily unique with respect to fixed content (Figure 7).

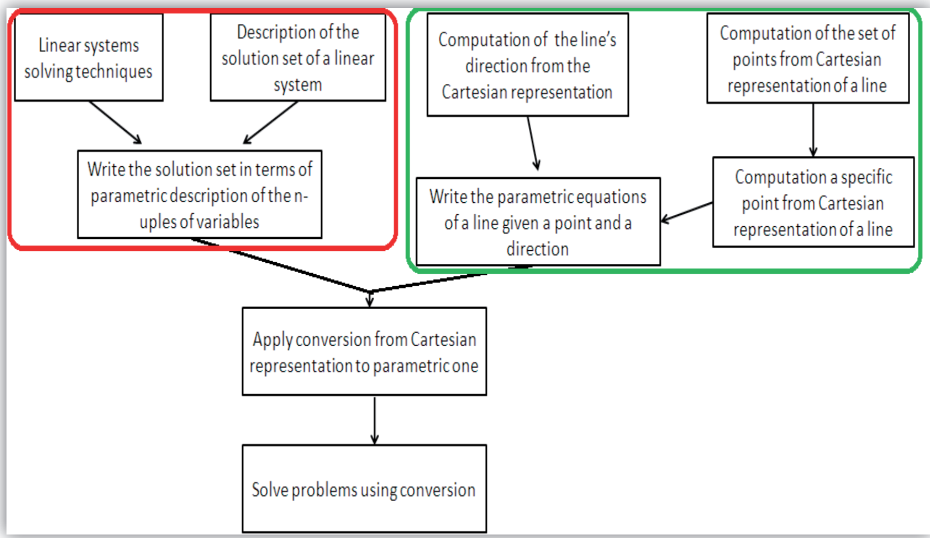

Figure 7. Example of algebraic (on the left) and geometric (on the right) genetic decomposition for a given competency.

Moving along a GD, the mechanism for practicing and constructing mathematical competency is described in terms of the following four elements (APOS):

- *Action*: a transformation generated as a reaction to external stimuli (physical or mental).
- *Process:* the interiorisation of the object, so that transformations can be mentally imagined.
- *Object:* the encapsulation of the process, due to reflections on operations applied to a particular process, making the individual aware of the process in its totality.
- *Schema:* objects and processes can be organized in a coherent collection, explicating the interconnections between them and giving rise to what is called a 'schema'. A schema represents an individual's knowledge of a competency and it is invoked in order to understand, deal with and face a perceived situation involving that competency.

Given a competency, its GD together with the related APOS give rise to a Learning Scenario (LS) suitable for a learner to practice and master such a competency.

Instructional treatment

Theoretical analysis indicates a specific LS to be fostered by instruction. This means designing instruction for a LA associated with a LS, which enables students to construct the appropriate actions, processes, objects and schemata. Such instruction can be described using a specific language for learning design (for instance, IMS-LD 2003) allowing the description of an activity workflow associated with the LS. These workflows include the definition of actions, processes, pedagogical strategies and specific environments comprising sets of LOs and services (forum, chat, calendar, virtual classroom, access to maths engines, etc.). The outcome of this phase will be one or more *templates* for a LA associated with a fixed LS. The *templates* also contain descriptive information in order to associate a LA with both a competency and to one or more concepts in an ontology (at knowledge and/or skill level).

Data collection/analysis

Once the instruction is implemented and experienced by students, observations and analysis of learning results in terms of theoretical expectations are needed. This means examining whether students have made the mental constructions predicted by the theoretical analysis or whether they have used alternative constructions. The data are used to validate the theoretical analysis and the consequent instruction treatment. Appropriate adjustments or a complete revision can be made.

6. How the new models work

From the above sections, we can sketch out the following Figure 8.

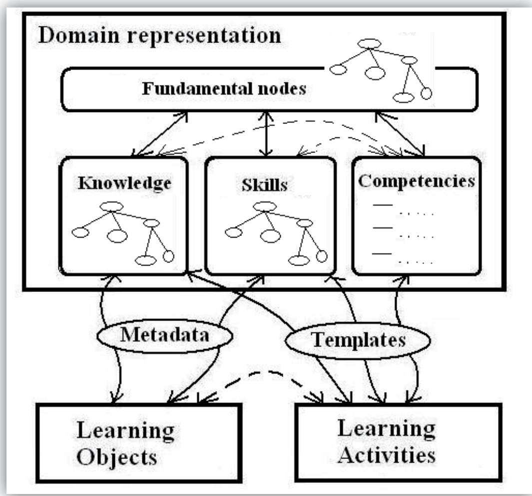

Figure 8. Model of learning experience generation.

Domain representation in the platform will consist first of all of an ontology on fundamental nodes, and then further levels of ontologies (knowledge and skills) and a database of competencies are devised.

Taking into consideration both guided and self-regulated learning, let us see how the new domain representation impacts on them. Concerning the former, the UoL corresponding to the standard course differs from the ones described in section 3.2 with respect to two aspects:

- The selection of target concepts can be specified at one or more ontology levels and the learning path will develop from the merging of the lists generated at each level; then the process continues as previously shown.
- The UoL will be enriched by engaging learners in LAs corresponding to selected competencies in the third level of domain representation (sections 4 and 5). The choice of LAs will be guided by the best match between a *template*'s descriptive information and a user profile.

Concerning self-regulated learning, the models are also able to meet learners' needs with respect to competencies (e.g., to learn proving statements). In this case, from among the available LAs, the platform selects the ones that best match the needs expressed by learners and, at the same time, that refer to concepts (in an ontology) already present in their cognitive states. In any event, a pre-test on such concepts can be done and a tailored UoL can be offered to each learner in order to bridge the gap if necessary.

7. Costs, benefits and feedback

In traditional teaching, teachers are, at one and the same time, the authors, tutors and evaluators of their courses. In an e-learning environment, we can explicitly distinguish the roles of author and tutor. Authors are collective subjects possessing all the skills required for the preparation of teaching materials in a digital context; they are not only experts with competencies in general and discipline-specific education, but also professionals with technical capabilities in ICTs, management and pedagogy. Tutors can be human or artificial agents to give students the right scaffolding needed to reach the desired educational goals. Teachers can assume one or more roles, including that of author, according to their expertise. For instance, in the case of our courses at the University of Salerno, teachers act as domain experts in Geometry or Calculus and have designed the related ontologies (by means of a user-friendly graphical tool, shown in Figure 2). They have also designed various LOs (from hypermedia to structured video and dynamic exercises with *Mathematica*[2]) and supervised their implementation by suitable technical staff.

While the work of an author in an e-learning context may seem tougher (since it also requires technical capabilities) than that of a teacher in traditional lectures, there are some considerable advantages, including:

2. www.wolfram.com

- Reusability: ontologies, LOs and *templates* constitute a repository available to all authors using the platform (not only to the owner).
- Continuous enrichment of the learning pool: this is a direct consequence of the previous item, as every teacher can take advantage of the others' work, thus benefitting from the chance to use much more material than they are individually capable of producing.
- Support for diversity in students' learning methods: the personalisation of teaching is not possible at undergraduate level, especially with large classes of freshers, but blended courses that combine face-to-face classes and distance mathematics instruction/learning can bridge the gap.
- Automatic learning tracking data: for both individuals and groups. Their analysis provides a great deal of information at the domain level (e.g., topics with intrinsic difficulty) and at learning level (e.g., basic shortcomings) so that adjustments can be made in the design/implementation of LOs and LAs.

Regarding students, we can make some considerations on the basis of our experience at the University of Salerno. Over the last few years, some mathematics courses at the University of Salerno have been IWT supported. Traditional classes have been supported by distance instruction, consisting of tailored UoLs (section 3.2) and teacher-driven cooperative or individual learning activities (whose formalisation, and the generalisation of the latter, has given rise to the model in section 5). Apart from the grades obtained in exams, we submitted questionnaires to students engaged in blended classes in order to investigate outcomes concerning meta-cognitive and non-cognitive levels. We essentially found that LAs have fostered a change in their working methods: going into depth as a standard practice, broadening perspectives, changing attitudes towards learning, focusing on relevant activities, organizing homework timetables and giving continuity to their work. Besides changing their working methods, they begin to grasp mathematical meanings and improve their ways to tackle problems, which were our main goals. Then their attitudes towards mathematics change (even those of individuals who are not usually successful at mathematics), thus initiating a productive learning process.

8. Future trends

We plan to continue our research on the knowledge-skill-competency model. Implementation on a platform requires details to be investigated, and integration with IWT algorithms for the automatic generation of personalised UoLs poses new open problems, such as:

- Investigation into tools useful for instructional treatment: it would be very interesting to have the chance to choose, at run-time, LOs involved in LAs, taking account of the assigned metadata.
- Investigation into the possibilities of LA interconnection with UoLs needed as prerequisites.

- Definition of competency assessment procedures according to the requirements of PISA (OECD, 2009), both in closed and open form (Albano, 2011b; Albano et al., 2008).
- Integration of the outcomes of competency open assessment to automatically update the UoLs and choose subsequent LAs.

9. Conclusions

In the context of mathematics e-learning, in this paper we have focused on competency learning. Assuming that competencies develop from learners' engagement in LAs, we have proposed a model apt to generate learning experiences, which can be further tailored to individual learners according to their user profiles. This model supplements the knowledge and skill models based on multi-graphs. All three models interact in order to generate a knowledge-skill-competencies model capable of creating and delivering personalised UoLs consisting of collections of LOs or LAs. The IWT platform has been used to validate the models in undergraduate courses. The outcomes show that students improve their ways of tackling mathematics problems or studying, while changing their attitudes towards mathematics.

References

ALBANO, G. (2011a). "Knowledge, Skills, Competencies: a Model for Mathematics E-Learning". In: R. Kwan; C. McNaught; P. Tsang; F. L. Wang; K. C. Li (eds). *Enhancing Learning Through Technology: International Conference, ICT 2011, Hong Kong, July 11-13. Proceedings (Communications in Computer and Information Science)*. CCIS 177. ISBN: 978-3-642-22382-2. Springer Heidelberg. Pages 214-225.

ALBANO, G. (2011b). "Mathematics education: teaching and learning opportunities in blended learning". In: A. Juan; A. Huertas; S. Trenholm; C. Steegmann (eds). *Teaching Mathematics Online: Emergent Technologies and Methodologies* [in press].

ALBANO, G.; FERRARI, P. L. (2008). "Integrating technology and research in mathematics education: the case of e-learning". In: F. J. García-Peñalvo (ed.): *Advances in E-Learning: Experiences and Methodologies*. IGI Global. Pages 132-148.

ALBANO, G.; GAETA, M.; RITROVATO, P. (2007). "IWT: an innovative solution for AGS e-Learning model". *International Journal of Knowledge and Learning*. Vol. 3, No 2/3, pages 209-224.

ARZARELLO, F.; ROBUTTI, O. (2002). *Matematica*. Brescia: La Scuola.

ASIALA, M.; BROWN, A.; DEVRIES, D. [et al.] (1996). "A Framework for Research and Curriculum Development in Undergraduate Mathematics Education". *CBMS Issues in Mathematics Education*. Vol. 6.

D'AMORE, B. (2000) "La complessità dell'educazione e della costruzione dei saperi". *Riforma e didattica*. No 4, pages 35-40.

GAETA, M.; ORCIUOLI, F.; RITROVATO, P. (2009) "Advanced Ontology Management System for Personalised e-Learning". *Knowledge-Based Systems*. Special issue. Vol. 22, Iss. 4, pages 292-301.

GODINO, J. (2002). "Competencia y comprensión matemática: ¿Qué son y cómo se consiguen?". *Uno*. Vol. 8, No 29, pages 9-19.

NISS, M. (2003). "Mathematical competencies and the learning of mathematics: The Danish KOM project". In: A. Gagatsis; S. Papastavridis (eds). *Proceedings of the 3rd Mediterranean Conference on Mathematical Education*. Athens: Hellenic Mathematical Society. Pages 115-124.

OECD (2009). *PISA 2009 Assessment Framework - Key Competencies in Reading, Mathematics and Science*. [Accessed: 28 January 2011].
<http://www.oecd.org/dataoecd/11/40/44455820.pdf>

SKEMP, R. (1976). "Relational understanding and instrumental understanding". *Mathematics Teaching*. No 77, pages 20-26.

WEINERT, F. (2001). "Concept of competence: a conceptual clarification". In: D. Rychen; L. Salgenik (eds). *Defining and electing key competencies*. Seattle, Toronto, Bern, Göttingen: Hogrefe & Huber Publishers.

WILEY, D. A. (2000). "Connecting learning objects to instructional design theory: A definition, a metaphor, and a taxonomy". In: D.A. Wiley (ed.). *The Instructional Use of Learning Objects*. [Accessed: 18 February 2010].
<http://reusability.org/read/chapters/wiley>

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Dossier "Mathematical e-learning"

ARTICLE

Activity Theory and e-Course Design: An Experience in Discrete Mathematics for Computer Science

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Abstract
The aim of this article is to present a distance e-learning experience of mathematics in higher education. The course is offered as a remedial program for master's degree students of Computer Science. It was designed to meet the particular needs of the students entering the master's degree program, as a response to the lack of understanding of logical language which was identified in several previous cohorts of students at CENIDET. The course addresses mathematical abilities of comprehensive functional use of logical language as a basic ability to be developed for later successful participation in the Master of Computer Science and also for later use in professional contexts of Computer Engineering. Eighteen students distributed throughout Mexico volunteered to participate under the guidance of one instructor. The techno-pedagogical design of the course is grounded on two theoretical approaches. Content-related instructional decisions are supported by different concepts of the second generation of Activity Theory. The concept of Orienting Basis of an Action was particularly useful to define the skills the students were expected to develop. Instructional decisions related to the participants' interaction are underpinned by Slavin's Team Accelerated Instruction model. We present the course structure in detail and provide some student interaction excerpts in order to illustrate their learning progress.

Keywords
e-learning, instructional design, higher education, discrete mathematics, activity theory, mathematical abilities

Teoría de la actividad y diseño de cursos virtuales: la enseñanza de matemáticas discretas en Ciencias de la Computación

Resumen
El objetivo de este estudio es presentar una experiencia de aprendizaje virtual a distancia en el ámbito de la enseñanza de las matemáticas en educación superior. El curso se ofrece como programa de apoyo para alumnos de un máster de Ciencias de la Computación y está específicamente diseñado para satisfacer las necesidades de los estudiantes que iniciaban dicho programa, particularmente la falta de comprensión del lenguaje lógico detectada en varias promociones anteriores de los alumnos del CENIDET. El curso tiene como objetivo el desarrollo de la habilidad de uso del lenguaje lógico, la cual es básica para cursar con éxito el máster de Ciencias de la Computación, así como para su posterior aplicación en contextos profesionales relacionados con la Ingeniería computacional. Dieciocho estudiantes distribuidos por todo México participaron voluntariamente en el estudio bajo la dirección de un tutor. El diseño tecnopedagógico del curso se basa en dos premisas teóricas. Las decisiones didácticas relacionadas con el contenido se fundamentan en varios conceptos derivados de la segunda generación de la Teoría de la Actividad (TA). El concepto de «base de orientación para la acción» ha sido particularmente útil para definir las habilidades que se esperaba que desarrollaran los estudiantes. Las decisiones didácticas relacionadas con la interacción de los participantes se basan en el modelo de enseñanza acelerada en equipo de Slavin. A continuación se expone detalladamente la estructura del curso y se presentan algunos extractos de la interacción de los estudiantes para ilustrar su proceso de aprendizaje.

Palabras clave
aprendizaje virtual, diseño didáctico, educación superior, matemática discreta, teoría de la actividad, habilidades matemáticas

1. Introduction

Working with formalized or semi-formalized statements is often a challenge for many students of mathematics. A typical coping strategy is to read the statements informally, avoiding mathematical formalism. Unfortunately, an important loss of mathematical knowledge happens by using this strategy. Various researchers have linked these difficulties with: i) the negation of mathematical statements (Antonini, 2001; Durand-Guerrier, 2004); ii) the translation (formalization) of natural-language statements into the formal language of First Order Logic (FOL) (Barker-Plummer, Cox, Dale & Etchemendy, 2008); and iii) the identification of the logical structure of mathematical statements (Selden & Selden, 1996).

In the field of Computer Science (CS), there have been recent proposals to include Formal Methods in the curriculum. Students are now expected to develop skills to read and write formal specifications for professional practice (Boca, Bowen & Duce, 2006). However, although many of them first encounter formalized or semi-formalized mathematics in Discrete Mathematics (DM) courses, their instructors often expect them to master FOL already. Hence, the students face difficulties in understanding and communicating new and complex concepts in their discipline with semi-formalized texts. Consequently, they need specific help to develop the skills to read mathematical texts in different settings. A good presentation of content does not suffice; therefore, remedial courses in higher education must work towards the explicit development of this ability (Merisotis & Phipps, 2000).

This paper describes the use of some elements of the second generation of Activity Theory (AT) to design an online remedial course. Particularly, the concept of Orienting Basis of an Action (OBA) was helpful to provide master's degree students with the necessary support in their learning processes. The online remedial course covers DM Preliminary Concepts (Logic, Set, Relations and Functions) for students entering a Master of CS in Mexico. In the following sections, we shall present the contextual background in detail, the theoretical assumptions and the consequences for instructional design. Some excerpts of the interactions occurring during the course will illustrate the students' learning progress.

2. The institutional context: mathematics teaching in Computer Science

Most DM courses offered to CS students follow a traditional model of mathematics teaching, such as: (1) concept definition; (2) theorem presentation; (3) demonstration; and (4) exercising (see Meyer, 2005, for example). Alternative courses are still the exception. Furthermore, they usually lack the theoretical underpinning of mathematical education (see, for example, Sutner, 2005).

Whether traditional or problem-based, all of these instructional proposals emphasize the accuracy of mathematical definitions. They all establish the definitions of content by means of logical language. In contrast, previous evaluation of the Mexican context (Ramírez, 1996; 2005) has repeatedly pointed to two comprehension *lacunae* among CS students: (a) translating mathematical language into natural language (and vice versa); and (b) analysing mathematical definitions. Hence, instructional programs of mathematics for CS students should address both topics. Regarding the latter topic in particular, students need to connect different representations of one concept, such as natural language, logical language, mathematical language and pictographic language.

3. Theoretical background: Activity Theory

AT allows mathematics educators to address all the above-mentioned deficiencies and requirements in DM online courses. Currently, there are three generations of AT (Engeström, 2000). The concepts defined by the first generation of AT – mediation, internalization and zone of proximal development (Vygotsky, 1988) – and those proposed by the third generation – learning by expanding, shared zone of proximal development (Engeström, 1987) and situated learning (Lave & Wenger, 1991) – are well established. In contrast, the development and applications of the second generation of AT are less recognised. Our instructional experience specifically draws on the second generation of AT. One of the basic elements of this approach is the accurate definition of the activity structure, through actions and operations (Leontiev, 1984). These concepts enable the study of human activity by facilitating the characterization of the notion of ability, which is a key element in our instructional proposal, both for the design of activities and learning materials, and for the analysis of advancements in learning. In the following subsections, we shall address the instructional decisions step by step, as guided by this theoretical framework.

3.1. The second generation of AT and mathematics teaching in higher education

Leontiev's concept of activity was used by Tallizina (1988), and later by Hernandez (1989) and Valverde (1990), among others, to describe mathematical abilities. For Leontiev, the activity appears as a refinement of the internalization concept and is a constituent element of the psychological subject in both its cognitive (awareness) and its affective and motivational aspects (personality). The activity orients the subject in the objective reality, transforming it into a form of subjectivity. That is, an activity is not just a reaction or a series of reactions; it is a system with structure, development, transitions and internal changes. An activity system produces actions and is, in turn, realized through actions. However, the activity is irreducible to particular actions. Each activity is always linked to a motive (either material or abstract), which responds to a need. The components of human activities are the actions performed by the individuals. The action has an operational aspect (how, by what means can we achieve the objective?), defined by the objective conditions required to achieve the goal of the activity. Activities, actions and operations are dynamic: they can change their 'level' within the macrostructure of the activity under certain conditions.

The design of a learning process departs from the psychological characterization of the activity in terms of its structural components: actions and operations. The educational interpretation of these components is expressed in terms of skills, and requires the mastery of a complex system of actions for self-regulation of the activity. The process of acquiring abilities involves the systematization of the actions they comprise. In turn, this process requires a conscious execution by the subject. The successful execution of the actions indicates the degree of skill development to perform the task. Hence, the subject must master the system of actions in order to fully develop the skill. In other words, we could argue that, for the teaching of mathematical text comprehension, it is essential to characterise the ability and identify the skills it comprises.

3.2. Designing the Orienting Bases of Action

The development of higher mental functions has a social origin (Vygotsky, 1988). This development happens in two separated stages: interpsychological and intrapsychological. Thus, development results from internalized actions. Galperin's Theory of the Stage-by-Stage Formation of Mental Activity (Galperin, 1969) is grounded on Vygotsky's premises and applies them to the instructional context. First, there is the stage of material activity, in which the learner needs to manipulate real objects, and embodied activity, in which the individual can handle models, diagrams and drawings, depending on the learner's age. Second, there is verbalization, when the student needs to repeat the sequence of operations aloud. By rewording, the action moves from the outside to the inside. Finally, the activity can take place at a wholly inner level, which implies thinking.

These evolutionary events can be tailored through the performance of certain guided actions. It is precisely that set of actions which will allow the student and the instructor to monitor and, if necessary, to correct each stage of assimilation. Galperin introduced the term 'orienting basis of an action' (OBA) to refer to the whole set of orienting elements by which the student is guided towards the successful execution of an action (also conceptualized as 'scaffolding' (Samaras & Gismondi, 1998).

In our study, we assume that the ability to read and understand mathematical texts comprises the following actions: (a) the translation of a mathematical statement into natural language, or vice versa; (b) the translation of the statement into the FOL language in order to reveal its structure; and (c) the

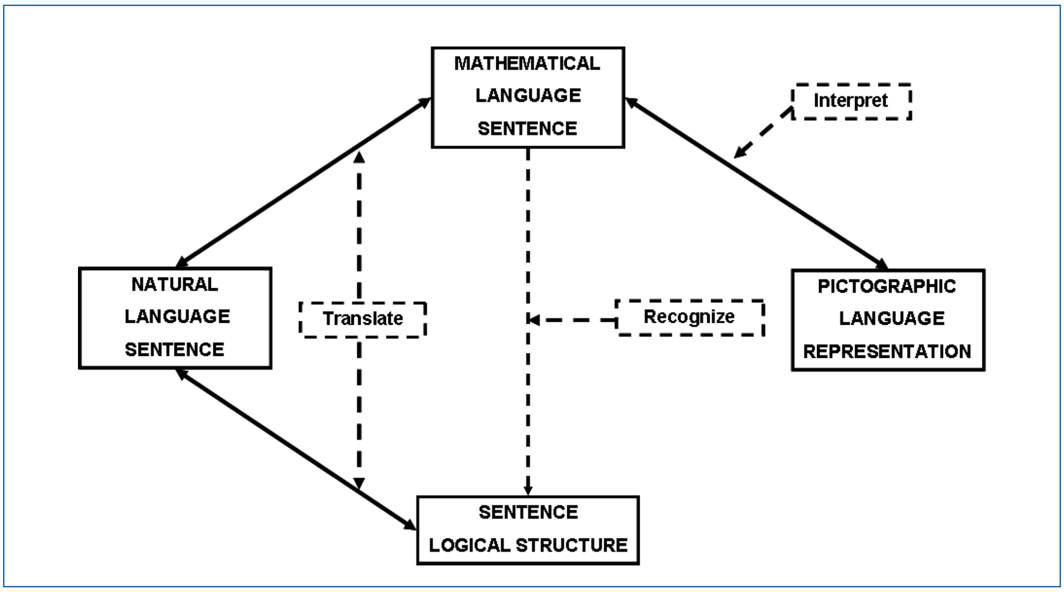

Figure 1: Ability system for the reading of mathematical texts.

representation of the statement in graphical language. To proceed with these steps and correctly interpret the mathematical statements, the students need to master both codes. This identification and characterization of the necessary skills for reading and comprehending mathematical texts provides a basis for designing and implementing online instructional processes (see Figure 1).

4. Addressing the challenges of instructional design

In line with AT, we took three axis elements for the instructional design: course objectives, contents and the scaffolds for the students (OBAs). We shall address each of these elements one by one further below. We shall then explain how these elements were implemented in a Learning Management System (LMS).

4.1. Course objectives

As a result of the problems identified in previous courses, we wanted the students to develop skills to identify and analyze the formal language (logical language and mathematical language itself) and thus represent mathematical concepts and their definitions. This main goal was divided into three objectives:

Students should...

a) Analyze and identify the FOL language in natural language.
b) Identify the mathematical language that is expressed through logical language, and the mathematical entities to which it relates.
c) Interpret definitions in mathematical language, both in formal and pictographic codes.

4.2. Content focus

The basic contents of DM traditional courses in higher education are *Propositional Logic, Predicate Logic, Sets, Relations and Functions*. Logic is usually taught on the basis of a deductive model of content presentation, aiming at demonstration, and using its own rules. In contrast, our course focused on the handling of FOL language and emphasized the process of translation of natural-language statements into logical and mathematical languages. The topic Reading Mathematical Texts was introduced after the Logic unit. For the translation from natural to mathematical language, students received a specific OBA.

The topics *Sets, Relations and Functions* had the following structure: first, the instructor presented a brief reading of the disciplinary area, where target mathematical concepts appeared in their usual contexts. After that, he presented the mathematical subject with standard texts. Thirdly, the students performed the exercise system for each topic, which included two main activities: (a) the analysis of definitions and (b) the use of the corresponding OBAs. Finally, the students received other additional readings of the disciplinary area, in which the target mathematical concepts appeared.

4.3. Orienting Bases of Action

We defined OBAs to help the students in their problem-solving processes. In this course, there were OBAs to translate statements: (a) from natural to propositional-logic language; (b) from natural to predicate language; (c) from natural to mathematical language, and vice versa. Finally, we proposed one OBA for (d) reading mathematical texts and for (e) analyzing the definition of mathematical concepts.

We offered the OBAs throughout the course, along with the material used by the students, introducing them by examples. In the unit Logic, OBAs were characterised and provided in order to develop the translation ability from natural language into FOL language. For the units *Sets, Relations and Functions*, an OBA was provided for the analysis of definitions. The following is an example of a partial implementation of an OBA for the analysis of definitions.

4.3.1. Example of an OBA

Initially, we offered the students step-by-step examples, performing the eight actions of the analysis: (1) differentiating between the expression of the definition in natural language and its expression in mathematical language; (2) identifying the mathematical entities in it; (3) giving examples of objects that both meet and do not meet the definition; (4) finding different ways to represent it; (5) identifying the underlying logical structure; (6) setting its negation; (7) finding the logical equivalence of the definition; and finally, (8) generalizing it.

The process presented to students as a model for using the OBA is outlined in Figures 2 and 3. In these Figures, the actions are listed in the left-hand column and possible responses are illustrated in the right-hand column.

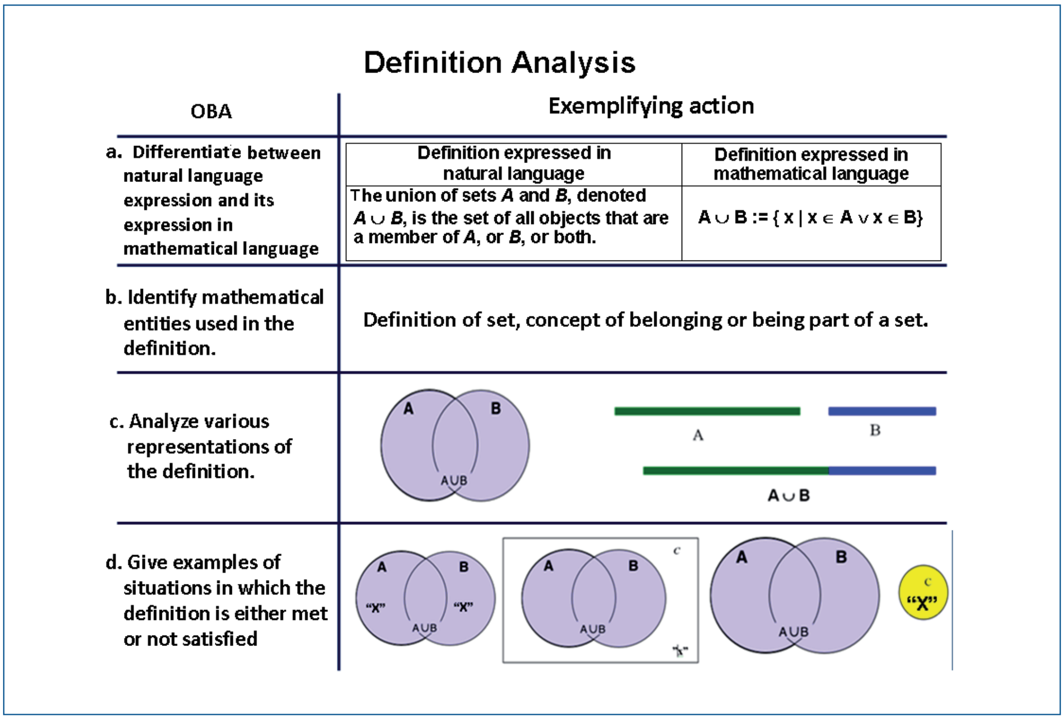

Figure 2: Actions #1 to #4 presented to students as an example for the analysis of a definition.

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Definition Analysis

OBA	Exemplifying action
e. Identify the definition's logical structure	If P(x) means that x belongs to A and Q(x) means that x belongs to B, the union definition of the logical structure of two sets is: $P(x) \vee Q(x)$.
f. Set the denial of the definition	From the definition's logical structure $P(x) \vee Q(x)$ we apply the logical negation $\neg(P(x) \vee Q(x))$, getting $\neg P(x) \wedge \neg Q(x)$. So the denial is $\neg(A \cup B)=\{x \mid x \notin A \wedge x \notin B\}$
g. Find the logical equivalences of the definition	In this case an equivalent expression to $P(x) \vee Q(x)$ would be unnatural, we could use $\neg(\neg P(x) \wedge \neg Q(x))$. How can we read this expression in natural language?
h. Generalizing the definition	The union of a finite number of sets is given by "successive unions" $[((A \cup B) \cup C) \cup D) \ldots \cup W] := \{x \mid x \in A \vee x \in B \vee x \in \ldots \vee x \in W\}$ That can be summarized $\bigcup_{\beta \in \mathrm{B}} A_\beta = \{x \in U \mid \exists \beta \in \mathrm{B} : x \in A_\beta\}$

Figure 3: Actions #5 to #8 presented to students as an example for the analysis of a definition.

This OBA assumes the partial development of the skills to translate statements from mathematical, pictorial and natural languages into each of the three codes. Performing the analysis of definitions provides students with fertile ground for developing the ability to read mathematical texts.

4.4. Techno-pedagogical design of the e-course

In e-learning, many drop-outs are caused by a lack of motivation (Juan, Huertas, Steegmann, Corcoles & Serrat, 2008); therefore, the instructional design of the courses is a key issue in the context of adult education. The term techno-pedagogical design refers to the instructional characteristics of a course as supported by technological devices (Mauri, Colomina & De Gispert, 2009). Indeed, the design of e-learning courses cannot be reduced to the traditional elements of the curriculum, i.e., objectives, content and learning activities and assessment. On the contrary, it must include a reasoned selection and planning of the technological tools that will be used throughout the course, together with a plan for the actual use of these tools and spaces by all participants. Hence, the techno-pedagogical design must include a careful planning of the interactions (instructor-students and among peers) that will be pursued throughout the course.

4.4.1. The LMS

For this course, we used Moodle (V.1.5.8) as the LMS. Moodle presents a flexible structure and leaves many choices open to the course designers and instructors. For example, it is possible to manage different spaces for diverse, flexible groups within the same course. This feature was particularly relevant in this course since it allowed for whole-class group interaction as well as small-group private spaces. The course administrator/instructor takes these decisions, according to the techno-pedagogical design. Furthermore, it allows the management of the course content in individual modules. In this course, we presented each of the five topics separately, in 'week-mode', all of which had the same recursive structure to help the students assume the participation norms.

4.4.2. Interaction design

For the selection and planning of technological tools, it is necessary to determine the interaction among students, and between students and instructor. We adapted the collaborative technique called Team Accelerated Instruction (TAI) (Slavin, 1994) to the Virtual Learning Environment (VLE). In keeping with this technique, the students have to perform three kinds of activities. First, the students must work independently with the learning materials. They are expected to read the course materials and solve the exercises and problems. The second step is for them to work in pairs in order to share and discuss solutions and difficulties with exercises and problems. For this purpose, the students have access to both a synchronous (chat) and an asynchronous (forum) private room on the online platform. The third interaction level covers the whole group. Again, both chat and forum tools support group interaction. The use of each of these spaces and tools is regulated by means of specific participation norms. Figure 4 shows a scheme for the organization of participants and content-learning materials.

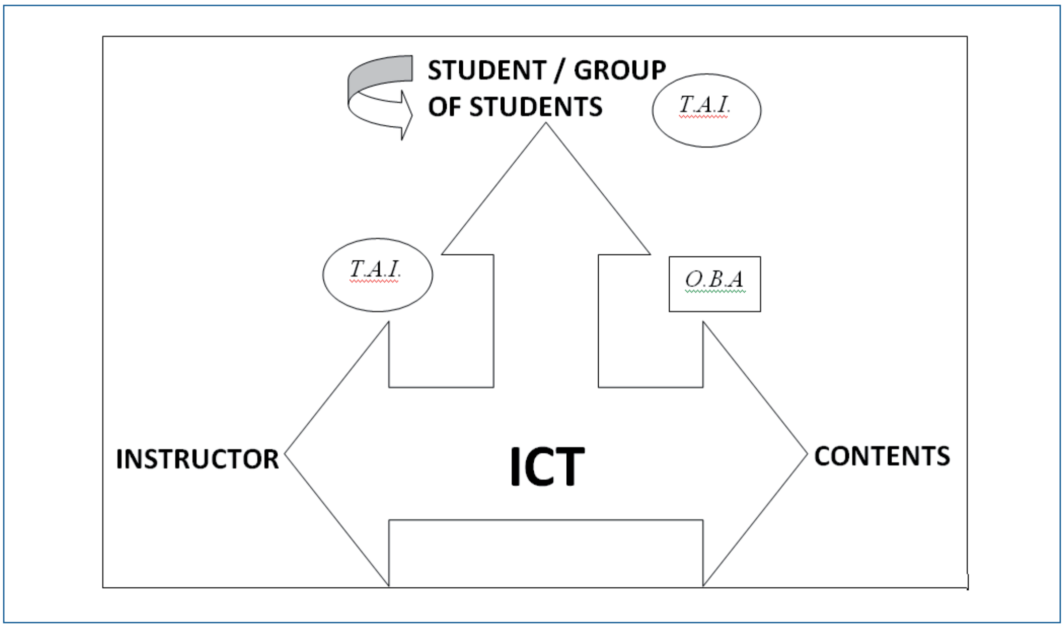


Figure 4: Techno-pedagogical course design.

Student-content interaction. Earlier editions of the course presented a technical difficulty while using chat and forum tools. The participants faced troubles when writing in logical and mathematical language. These problems had been reported previously in similar studies (Smith, Ferguson & Gupta, 2004). Thus, to facilitate mathematical communication, we added an HTML editor with a mathematical equation editor (WIRIS, V.2.1.26) to the chat device (Juárez & Ramírez, 2010). Figures 5 and 6 show the equation editor and some examples of how it can be used.

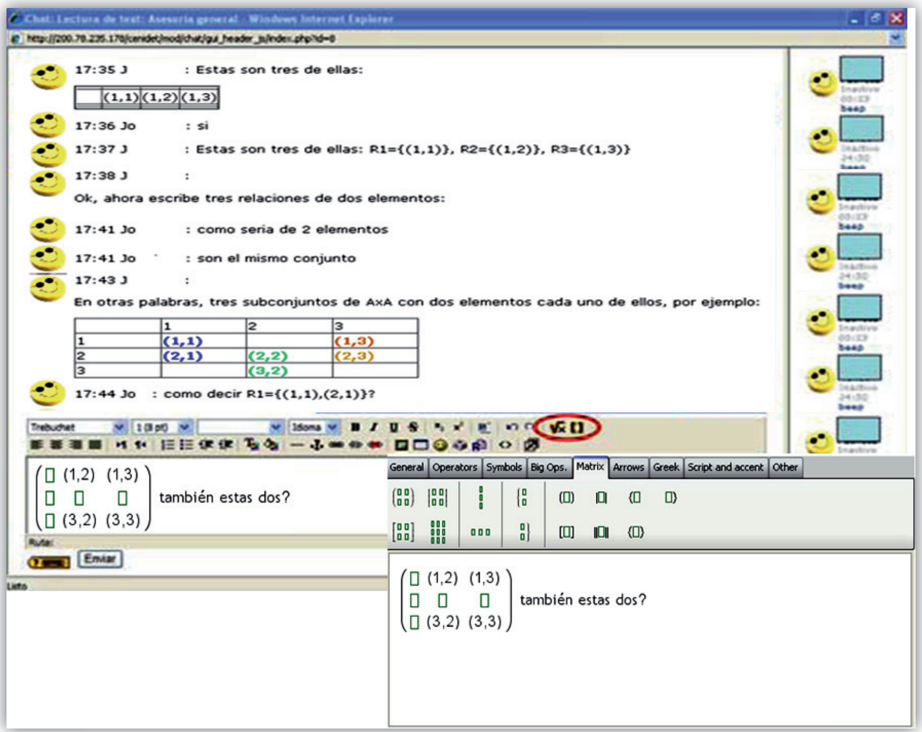


Figure 5: Example of the equation editor and its use in synchronous interaction.

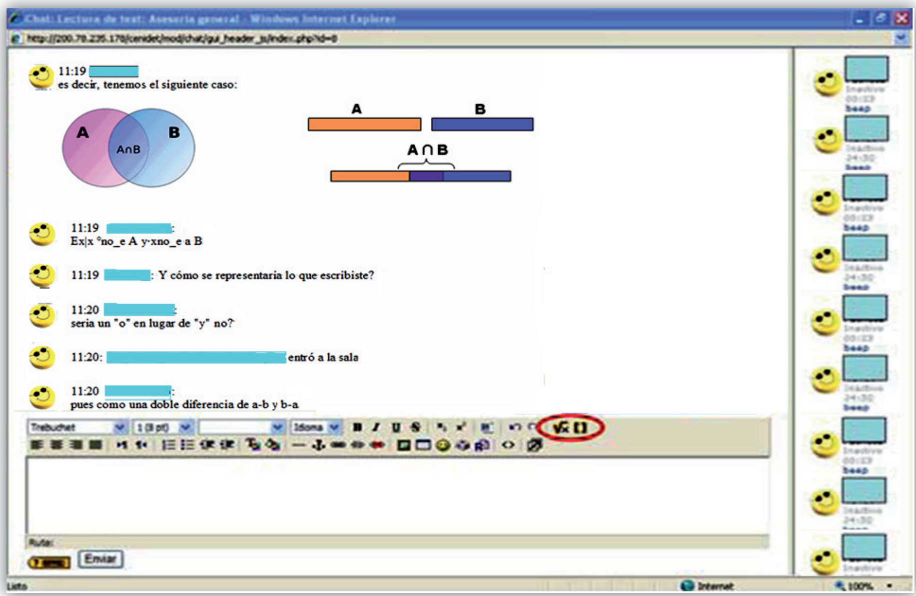


Figure 6: Using WIRIS for online pictorial representation.

Student-instructor interaction. The main area of the course had three communication spaces. First, there was a forum for small-group discussion. This forum provided an asynchronous space to facilitate continuity of discussions and feedback to students by the instructor. Second, there were two chat rooms for synchronous interaction, serving two purposes: there was a first chat room for organized whole-class discussion to resolve doubts under the instructor's guidance, and a second one for technical support.

Student-student interaction. Student-student interaction was designed to occur in pairs and was facilitated by means of different tools. First, there was a chat room for synchronous interaction; second, a wiki for joint resolution of mathematical problems; and third, a database to share results and reflections. Each pair of students could freely decide which tool to use. The group spaces were private to each pair of students; only the instructor could access all the small-group spaces. Hence, he was able to check or participate in the students' interaction, as would be the case in face-to-face TAI situations.

4.4.3. Course structure

The course lasted five weeks, from July to August 2008. A group of 18 students voluntarily accepted to enrol on the master's degree course in CS at CENIDET. The students were CS Engineers from several states across Mexico. The instructor had experience in conventional courses on the subject; in addition, he was familiar with basic technological tools and had participated in the course design.

The course included five units (*Logic and Mathematical Language, Sets, Relations, Functions and Applications*), one per week. The students worked together in pairs, following the TAI model previously presented. If doubts remained after peer interaction, they were brought to the whole-class forum or the whole-class chat.

After each week, the students carried out a self-assessment using a feedback sheet provided by the instructor as a model of resolution. The students had to compare the model with their own resolution in order to identify deviations, strengths and weaknesses. This self-assessment was not graded. The instructor conducted weekly sessions of two hours to offer guidance and clarify doubts. He gave feedback and interacted with the students both synchronously (whole-class chat room) and asynchronously (whole-class forum). Strict turn-taking was established in order to facilitate synchronous interaction in the whole-class chat room. Each student pair had a particular interaction turn with the instructor for 20 minutes. The other participants attended the chat session as observers; they had the chance to 'listen' until the change of turn. This instructional design is presented in more detail in an earlier publication (Remesal, Juárez & Ramírez, 2011).

5. Results: evidence of ability development through the use of Orienting Bases of Action

In order to assess the development of the students' abilities, we performed an interpretive analysis of the following discursive aspects (Lacity & Janson, 1994; Willig, 2004):

1. The students' answers to the exercises.
2. The questions asked in the forum and chat.
3. The comments made in student-student interaction.
4. The results of the students' weekly self-assessment.

We shall now go on to present the specific results of the analysis of data sources #1 to #3, with the purpose of illustrating rather than providing an exhaustive account.

In the following sequence, we can observe an example of a student's development of abilities, that is, of her internalization process (translated from the original language, participants are quoted by pseudonyms). First, we can see how Lois begins the analysis of the definition by indicating what definition she used for inverse functions; then she indicates her doubts regarding its logical structure. She finally tells Mary how she interprets the mathematical definition in natural language. Her explanations and doubts show the acquisition of a skill to organize definitions according to the model shown in the OBA. In this asynchronous interaction, Mary's answer shows how she partially performs the definition analysis. She starts with the definition in natural language and then in mathematical language. Mary provides more elements about what she understands as the logical structure of the definition. She concludes by commenting on the types of properties that a function must satisfy in order to have an inverse function. Despite her analysis, she is unsure about the correctness of the logical structure she proposes:

[Small-group forum. Author *Lois*. 08/02 08:59]

"Hello Mary,

What definition in mathematical language did you use for an inverse function? I used the following one, but I'm not sure what its logical structure is:

$$f^{-1}:B[A \mid x = f^{-1}(y),$$

which in natural language is: a function is inverse is any rule or correspondence that allows us to get the values of

$$x \in A \text{ from the values of } y \in B$$

Do you have any other idea?"

[Small-group forum. Author *Mary*. 08/02 11:27]

"Hello Lois! Good morning,

Well, let's see if I can help you, OK? Look, my definition of natural language came out like this....

Be A in B and B in A an inverse function, if and only if the function is bijective, that is, a one to one correspondence and above the co-domain.

$$f \Leftrightarrow f^{-1}$$

in mathematical language...

It's an inverse or invertible function.

$$f = f^{-1} \text{ If,}$$
$$f: A \rightarrow B \text{ and } f^{-1}: B \rightarrow A \text{ if,}$$
$$f: A \rightarrow B \ (\forall x \, \exists y \wedge \forall y \, \exists x)$$

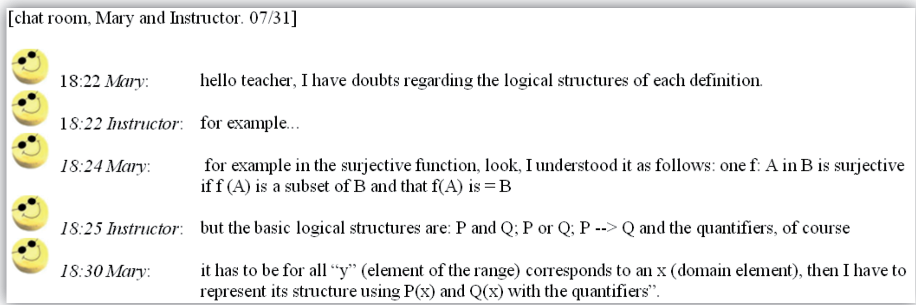
and in its logical structure, I'm not quite sure, but look at what I got…

P(x) ----> Q(x) and Q(x) ----> P(x)

Well, that's what I think, look, because reading the definition, it says that in order for a function to be inverse, you first need to know that it's a function, secondly, check if it has the properties of injective and surjective in order to say that $f: A \rightarrow B$ and $f^{-1}: B \rightarrow A$.

Well, girl, I hope this was helpful,... if it wasn't, tell me and we can talk, I may be wrong and that way we can clear things up, OK?"

Through peer interaction, by sharing doubts and efforts to grasp the OBA in order to handle mathematical content in the multiple communication spaces of the course, the students proved to be progressively internalizing these abilities. In the last units of this course, most students formed definitions according to the OBA through the following actions: first, negating the function; second, translating the function from mathematical language to natural language and vice versa; third, representing the different forms of the function; and finally, representing the logical structure.



[chat room, Mary and Instructor. 07/31]

18:22 *Mary*:	hello teacher, I have doubts regarding the logical structures of each definition.
18:22 *Instructor*:	for example...
18:24 *Mary*:	for example in the surjective function, look, I understood it as follows: one f: A in B is surjective if f (A) is a subset of B and that f(A) is = B
18:25 *Instructor*:	but the basic logical structures are: P and Q; P or Q; P --> Q and the quantifiers, of course
18:30 *Mary*:	it has to be for all "y" (element of the range) corresponds to an x (domain element), then I have to represent its structure using P(x) and Q(x) with the quantifiers".

Figure 7: Example of first steps of OBA use in a chat session.

For instance, the following intervention by Cinthya (Figure 7) shows how she explicitly states the first step for the OBA: "First is the analysis of the definitions. Please, tell me if I'm doing OK." At certain times, the instructor intervened actively to remind the students of the actions that structured the OBAs and guided them to establish a link to the learning content. In the following sequence, for example, the student shows some early recognition of the logical structure of a statement. Thus, the instructor intervenes to remind her of one of the scaffolding actions related to the translation of statements and the recognition of their logical structure.

After the instructor's indications on basic logical structures, Mary remembers the need to use the predicates and quantifiers. Feedback and scaffolding for solving exercises was not only provided by the instructor; at times, other students who participated in chat sessions also contributed to this assistance intervention.

In addition to the chat and forum interaction, the weekly quizzes that the students performed for each unit also informed on the skills development by means of the OBAs, For example, Figure 8 shows part of a student's answer to the first question the third weekly quiz. In this quiz, the student begins rewriting the entire OBA for definition analysis. Then he solves the exercise one step at a time.

Definition Analisys: Reflective Relation

SAGS
Team Lambda

For the definition analysis seven steps should be followed:

a) Distinguish between the definition expressed in natural language and its expression in mathematical language.
b) Think about the mathematical entity or entities referred to in the definition.
c) Analyze various definition´s representations.
d) Give examples of situations that either meet the definition or no meet that definition.
e) Identify the logical structure of the definition.
f) Establish the negation of the definition.
g) Find logical equivalences of the definition.

a) Distinguish between the definition expressed in natural language and their expression in mathematical language.

Natural Language Definition	Mathematical Language Definition
*The relation R is reflexive if every element of set A is related to itself. *A relation **R** on a set **A** is reflexive if any element of **A** relates to himself	Let A be any not empty set. Let R be a relation on A. R is reflexive if and only if $\forall x\ (x \in A \Rightarrow x\ R\ x)$

To complete this step, we must think other ways to say that a Relation is Reflexive and express it in mathematical language.

"R is a reflexive relation on A if and only if all elements of A are related to themselves via R"

RR= is the Set of Reflexive Relations on A.
R = a Relation
$R \in RR \leftrightarrow \forall\ x\ (x \in A \rightarrow x\ R\ x)$

Figure 8: Using the OBA to analyze a definition.

The student's response shows the first step of the definition analysis, expressed in both natural language and mathematical language. The student began his analysis by writing the expressions in both languages in a table. He concluded this step by proposing a different way of expressing the definition in natural language and its corresponding formalization in mathematical language. The student was able to propose his own way of describing the reflexive relationship concept and formalizing it in mathematical language. These actions show how the student used the OBA, and hence, they bear witness to his mastery of the targeted skill to translate a sentence expressed in natural language into mathematical language.

6. Conclusions: assessment of the course design

The need for remedial courses to promote Mexican students' successful participation in the CEDINET master's degree program has been confirmed in recent decades (Ramírez, 1996; 2005). In face-to-face educational situations, the development of mathematical skills is a complex teaching and learning task. Consequently, re-locating it in the virtual context is a risky business in its own right. Particularly, the instructional design and its implementation in an LMS constitute a fundamental challenge to higher education instructors. In this course, the techno-pedagogical design allowed the participants' interaction within the system to be anticipated by promoting student-student synchronous and asynchronous interaction followed by whole-class student-instructor highly structured synchronous interaction. On the one hand, this was pedagogically enabled by the TAI model. On the other, it was technologically facilitated by the flexibility of the LMS and the incorporation of the WIRIS application.

However, and most importantly, the online instructional design presented in this article strongly suggests that the second generation of AT provides useful theoretical elements for promoting the development of the pursued abilities by means of virtual tools. Based on AT, it was possible to define the course objectives in terms of skills, knowledge and application conditions. This in turn allowed the focus to be placed on skills development, using mathematical content as a means, in contrast to traditional approaches of teaching mathematics in higher education, which often focus on content presentation.

In earlier works, we reported the positive assessment that all the participants made of the course (Remesal, 2008). After analyzing the participants' interactions on the virtual platform with regard to undertaking the translation exercises following the OBAs provided, we evaluate the course design positively with respect to three important teaching aspects. First, regarding the content sequence, starting with logical-language proficiency and moving towards an understanding of semi-formalized mathematical texts appeared to be a highly appropriate strategy to facilitate the development of the target abilities. Second, the interaction structure and norms had three positive effects: (1) they allowed the resolution of exercises; (2) they fostered the appropriation of content and the development of abilities; and (3) they encouraged social interaction between pairs of students at a distance. And third, the incorporation of specific software (WIRIS) helped the participants to overcome difficulties in handling algorithmic and pictographic codes in virtual written communication.

Nevertheless, the insufficient duration of the course poses a clear limitation for the full development of the intended abilities, since the development of skills requires gradual practice; indeed, five weeks is too short a time span. In future editions of this course, a longer term (up to eight weeks) should be considered. In addition, we are looking at three possible directions for the next research and instructional steps. First, a longitudinal study is needed to identify how students use the OBA for the analysis of definitions on the Master of CS after the remedial course. This longitudinal project would facilitate the assessment of the remedial course. Second, we intend to expand the remedial course to other related content, such as *Modal Logic* and *Dynamic Logic* of the master's degree program. Finally, audio and videoconferencing tools will be added to upcoming editions in order to establish whether interaction is enhanced through their use.

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References

ANTONINI, S. (2001). "Negation in mathematics: obstacles emerging from an exploratory study". *Proceedings of the 25th PME Conference*. University of Utrecht. Pages 49-56.

BARKER-PLUMMER, D.; COX, R.; DALE, R.; ETCHEMENDY, J. (2008). "An empirical study of errors in translating natural language into logic". *Proceedings of the 30th Annual Meeting of the Cognitive Science Society/CogSci*. Pages 505-510.

BOCA, P.; BOWEN, J. P.; DUCE, A. (eds) (2006). *Teaching Formal Methods: Practice and Experience*. Electronic Workshops in Computing (eWiC) series. London: BCS London Office.

DURAND-GUERRIER, V. (2003). "Which notion of implication is the right one? From logical considerations to a didactic perspective". *Educational Studies in Mathematics*. Vol. 53, pages 5-34.

DURAND-GUERRIER, V.; BEN-KILANI, I. (2004). "Négation grammaticale versus négation logique dans l'apprentissage des mathématiques. Exemple dans l'enseignement secondaire Tunisien". *Les Cahiers du Français* Contemporain. Vol. 9, pages 29-55.

ENGESTRÖM, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki: Orienta-Konsultit Oy.

ENGESTRÖM, Y. (2000). "Activity theory and the social construction of knowledge: A story of four umpires". *Organization*. Vol. 7, No 2, pages 301-310.

GALPERIN, Y. (1969). "Stages in the development of mental acts". In: COLE, M.; MALTZMAN, I. (eds). *A handbook of contemporary Soviet psychology*. New York: Basic Books. Pages 249–273.

JUAN, A. A.; HUERTAS, A.; STEEGMANN, C.; CORCOLES, C.; SERRAT, C. (2008). "Mathematical e-learning: state of the art and experiences at the Open University of Catalonia". *International Journal of Mathematical Education in Science and Technology*. Vol. 39, No 4, pages 455-471.

JUÁREZ, M.; RAMÍREZ, J. L. (2010). "Colaborar para aprender y enseñar matemáticas online". *Didac*. Nos 56-57, pages 71-75.

JUNGK, W. (1981). *Conferencias sobre la enseñanza de la matemática*. Havana: Ministerio de Educación.

LAVE, J. & WENGER, E. (1991). *Situated Learning: legitimate peripheral participation*. Cambridge: Cambridge University Press.

LACITY, M. C.; JANSON, M. A. (1994). "Understanding qualitative data. A framework for text analysis methods". *Journal of Management Information Systems*. Vol. 11, No 2, pages 137-155.

LEONTIEV, A. N. (1984). *Actividad, Conciencia y Personalidad*. Mexico: Cartago.

MAURI, T.; COLOMINA, R.; DE GISPERT, I. (2009). "Diseño de propuestas docentes con TIC en la enseñanza superior: nuevos retos y principios de calidad desde una perspectiva socioconstructivista". *Revista de Educación*. Vol. 348, pages 377-399.

MERISOTIS, J. P.; PHIPPS, R. A. (2000). "Remedial education in colleges and universities. What's really going on?" *The Review of Higher Education*. Vol. 4, No 1, pages 67-85.

MEYER, A.; RUBINFELD, R. (2005). *Mathematics for computer science* [course notes]. *MIT OPENCOURSEWARE*. Massachusetts Institute of Technology.
<http://ocw.mit.edu/OcwWeb/Electrical-Engineering-and-Computer-Science/6-042JFall-2005/CourseHome/index.htm>

RAMÍREZ, J. L. (1996). *Reporte del proyecto: "Estructuración de una metodología para la enseñanza de las matemáticas discretas para la maestría en ciencias de la computación en el CENIDET"* [internal report, course notes]. CENIDET. Departamento de Desarrollo Académico.

RAMÍREZ, J. L. (2005). *Reporte del proyecto: "Identificación de dificultades en los cursos de matemáticas de los programas de maestría del CENIDET"* [internal report]. CENIDET. Departamento de Desarrollo Académico.

REMESAL, A. (2008). *Lectura de textos semi-formalizados de matemáticas para computación. Informe técnico de evaluación* [internal report]. CENIDET/Universidad de Barcelona.

REMESAL, A.; JUÁREZ, M.; RAMÍREZ, J. L. (2011). "Technopedagogical design versus reality in an interinstitutional online remedial course" [paper]. *EARLI 2011 Education for a Global Networked Society*.
<http://aremor.wordpress.com/publications-2/>

SAMARAS, A. P.; GISMONDI, S. (1998). "Scaffolds in the field: Vygotskian interpretation in a teacher education program". *Teaching and Teacher Education*. Vol. 14, No 7, pages 715-733.

SELDEN, A.; SELDEN, J. (1996). "The role of logic in the validation of mathematical proofs". *Proceedings of The DIMACS Symposium on Teaching Logic and Reasoning in an Illogical World*. Rutgers University.

SLAVIN, R. E. (1994). *Cooperative Learning. Theory, Research, and Practice*. Boston: Allyn and Bacon.

SMITH, G. G.; FERGUSON, D.; GUPTA, S. (2004). "Diagrams and math notation in e-learning: Growing pains of a new generation". *International Journal of Mathematical Education in Science and Technology*. Vol. 35, No 1, pages 681-695.

SUTNER, K. (2005). *CDM: Teaching discrete mathematics to computer science majors.* Carnegie Mellon University.
<http://www.cs.cmu.edu/%7Esutner/papers/jeric.pdf>

TALLIZINA, N. F. (1988). *Los fundamentos de la educación superior*. Mexico: UAM-Ángeles Editores.

VALVERDE, L. (1990). *Un método para contribuir a desarrollar la habilidad fundamentar-demostrar una proposición matemática*. [Unpublished doctoral thesis]. Universidad de la Habana.

VYGOTSKY, L. S. (1988). *El desarrollo de los procesos psicológicos superiores*. Barcelona: Grijalbo.

WILLIG, C. (2004). *Introducing qualitative research in psychology. Adventures in theory and method.* Philadelphia: Open University Press.

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ARTICLE

Distance Training of Mathematics Teachers: The *EarlyStatistics* Experience

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Abstract

The affordances offered by modern Internet technologies provide new opportunities for the pre-service and in-service training of mathematics teachers, making it possible to overcome the restrictions of shrinking resources and geographical locations, and to offer, in a cost-effective and non-disruptive way, high-quality learning experiences to geographically dispersed teachers. This article focuses on how information and communication tools made available online could be exploited effectively to help improve the quality and efficiency of teacher training in statistics education. First, it describes the main pedagogical issues and challenges underlying distance education in general, and online teacher training in particular. Then, it provides an overview of *EarlyStatistics*, an online professional development course in statistics education targeting European elementary and middle school teachers, and the main lessons learned from the pilot delivery of it. The article concludes with some instructional implications.

Keywords
statistics education, e-learning, blended learning, teacher training

Formación a distancia para profesores de matemáticas: la experiencia de EarlyStatistics

Resumen
Las potencialidades que ofrecen las modernas tecnologías de internet brindan nuevas oportunidades a la formación inicial y permanente del profesorado de matemáticas, que permiten superar las limitaciones impuestas por recursos cada vez más escasos y por la ubicación geográfica, y que para este colectivo geográficamente disperso significan el acceso a un aprendizaje de calidad, económico y compatible con el resto de actividades. Este artículo se centra en cómo aprovechar eficazmente las herramientas de comunicación e información disponibles en línea para mejorar la calidad y la eficiencia de la formación del profesorado en la educación de estadística. En primer lugar, describimos los principales problemas y retos pedagógicos de la educación a distancia en general, y de la formación de profesorado en línea en particular. A continuación, ofrecemos una visión general de EarlyStatistics, un curso virtual de desarrollo profesional para la educación estadística dirigido al profesorado de educación primaria y primeros cursos de secundaria (de 6 a 14 años), y las principales conclusiones derivadas de la edición piloto del curso. Concluyen el artículo algunas sugerencias educativas.

Palabras clave
enseñanza de estadística, aprendizaje virtual, aprendizaje mixto, formación del profesorado

1. Introduction

In recent years, it has been recognized that for mathematics teacher training to become more effective in producing real changes to classroom practices, it ought to promote continuous, professional development opportunities that are cumulative and sustained over the career of a teacher (Joubert, 2009). The financial and logistic difficulties of engaging teachers in face-to-face training, as well as the need for professional development that can fit with teachers' busy schedules and can draw on powerful resources often not available locally, have encouraged the creation of online professional development programs for teachers (Dede, 2006).

This article analyses the possibilities of information and communication tools made available by modern Internet technologies to improve the quality of initial and in-service teacher training in statistics education. First, it discusses the main pedagogical issues and challenges underlying distance education in general, and online teacher training in particular. Then, it provides an overview of *EarlyStatistics*, a European Union-funded program that has utilized distance education to offer teacher training in statistics education. The article concludes with some implications for distance teacher training.

2. Distance education: main pedagogical perspectives and challenges

Educational institutions at all levels, including leading research universities, are becoming increasingly involved in distance education initiatives. Online course delivery has become common in a wide variety of disciplines, including mathematics and statistics, and this expansion is likely to continue, owing to ever-greater access to the Internet and emphasis on lifelong learning.

Several advantages associated with distance education have been identified in the literature. Distance education offers flexibility and convenience, allowing learners to determine their own place, pace, time and content of study. Further, the distance option may give students the opportunity to take courses from prominent experts in their field of study (Evans, 2007). Moreover, from the viewpoint of statistics education, network-based training creates some unique opportunities for enhancing statistics instruction. The Internet offers a vast array of tools and resources that can be used for a better understanding of statistical concepts. Interactive Java-applets and virtual statistical laboratory experiments, for example, allow for the visualization of statistical ideas and hands-on simulations with a high pedagogical potential (Vermeire, 2002). Several statistics instructors mention using technological tools and resources in their online courses (e.g., Everson, 2008).

In spite of the undisputed benefits and proliferation in recent years of online programs, concerns remain about their quality, as research suggests that the effectiveness of distance education is variable and inconsistent (Evans, 2007). While most studies indicate that students taking courses with an online component have similar achievement and satisfaction levels compared to students in traditional, face-to-face classrooms (Dutton, 2005), there is growing evidence of many web-based distance-learning courses failing to meet the expectations raised.

Early attempts at Internet-based instruction assumed that setting up an attractive website with interesting online and multimedia applications was sufficient for learning to take place. It is now recognized that the level of success of a distance-learning course is determined by multiple factors. Elements in the design of a web-based course such as the content and structure of the course, the presentation of the online materials and the amount of interaction between instructors and learners – as well as among learners themselves – are important factors affecting students' learning and attitudes (Tudor, 2006). Another important criterion of the level of success of web-based statistical training is the extent to which instruction allows learners to experience the practice of statistics and to apply statistical tools in order to tackle real-life problems (Vermeire, 2002).

In addition to the general issues and considerations regarding distance education in statistics, the distance training of statistics teachers poses special challenges. For example, one of the main challenges for developers of online teacher training programs is how best to take advantage of the variety social networking tools and technologies now available in order to foster the creation of online communities of teaching practitioners as vehicles for teacher learning and development. Research studies in this area indicate that online communities of practice are indeed a promising model for both pre-service and in-service mathematics teacher training (e.g., Cady, 2009). They have enormous potential to support the professional development of teachers by placing educators at the center of their learning, thus promoting their independence and self-directed learning. Online communities of practice facilitate not only communication, but also the collaborative finding, shaping and sharing of knowledge. At the same time, existing research highlights several difficulties in building and maintaining online communities involving shared professional learning.

Despite the early enthusiasm and encouragement of participants, many online communities of practice fail to thrive (Riverin, 2007). For example, after examining 28 studies, Zhao (2001) reported that there was little conclusive evidence to demonstrate the effective use of reflective online communities of practice. Other studies (e.g., McGraw, 2007) raise several issues that consistently create challenges for community building among participating teachers and for sustainability, including barriers to access, usability, sociability, lack of time to spend in online discussions, and language. In statistics education, while it is well-documented in the literature that incorporating discussion and active learning into the statistics classroom can help students learn to think and reason about statistical concepts, it has proved challenging to bring these important learning approaches to an online course (Everson, 2008).

Gould (2005), in their first offering of INSPIRE, a distance education professional development course targeting new secondary school statistics teachers in the U.S., which had community building as one of its main objectives, experienced disappointment, with a much lower than anticipated level of student-student interaction. A more successful example of a program adopting a community-building approach to the distance training of statistics teachers is Becoming a Teacher of Statistics, an online graduate-level course offered by the University of Minnesota that prepares teachers of introductory statistics at college and high school levels (Garfield, 2009). While originally delivered in a face-to-face setting, the course was subsequently converted into an online course to make it accessible to a wider variety of pre-service and in-service teachers. The first online version of the course was offered in spring 2008, with very encouraging outcomes. Evaluation of the course indicated that it was equally successful, and it provided students with parallel experiences to those in the face-to-face class.

3. Experiences of distance teacher training in Europe

In modern, information-based society, statistical concepts are occupying an increasingly important role in mathematics curricula across Europe. The subject, however, has been introduced into mainstream mathematics curricula without adequate attention being paid to teachers' professional development. There is substantial evidence of poor understanding and insufficient preparation to teach statistical concepts among many pre-service and practicing teachers (e.g., Espinel, 2008).

In this section, we provide a brief description of the main experiences gained from implementing the European Union-funded program *EarlyStatistics:* Enhancing the Teaching and Learning of Early Statistical Reasoning in European Schools (226573-CP-1-2005-1-CY-COMENIUS-C21). *EarlyStatistics* has exploited the affordances offered by Open and Distance Learning (ODL) technologies to improve the quality of statistics instruction offered in European schools. The project consortium, comprising five higher education institutions in four countries (Cyprus, Greece, Norway and Spain) developed, pilot tested and is currently offering an online professional development course targeting elementary and middle school mathematics teachers across Europe. The course, which is the first of its kind in Europe, aims to help teachers improve their pedagogical and content knowledge of statistics through exposure to innovative learning methodologies and resources, and cross-cultural exchange of experiences and ideas.

Before being offered to the European educational community, the *EarlyStatistics* course and its accompanying resources were pilot tested locally in three of the partner countries (Cyprus, Greece and Spain). Fourteen teachers participated in the pilot delivery. In order to evaluate the applicability and success of the course, there was also follow-up classroom experimentation. Participating teachers developed and delivered teaching episodes integrating the use of the course tools and resources provided to them. The course was revised based on feedback received from the pilot delivery, and then entered into the European Union Lifelong Learning Training Database for European-wide recruitment. It is offered to the European educational community as a Comenius in-service training course targeting elementary and middle school mathematics teachers. The course has already been offered twice. The consortium intends to continue offering the course in subsequent years, thus facilitating access to larger numbers of mathematics teachers involved in statistics education.

Presented below are an overview of the *EarlyStatistics* course design and a synopsis of the main findings from the pilot delivery of the course.

Design of the *EarlyStatistics* course

EarlyStatistics course content and structure

The *EarlyStatistics* course design focuses on participatory and collaborative learning. Teachers enhance their knowledge about statistics and its pedagogy through hands-on and computer-based practice, experimentation, intensive use of simulations and visualizations, feedback from one another and reflection. Then, being actual practitioners, they apply what they learn to a real classroom setting.

The *EarlyStatistics* course lasts for 13 weeks and is made up of six Modules. In Modules 1-3 (Weeks 1-6), the focus is on enriching the participants' statistical content and pedagogical knowledge by exposing them to similar kinds of learning situations, technologies and curricula to those they should employ in their own classrooms. The conceptual "Framework for Teaching Statistics within the K-12 Mathematics Curriculum" (Franklin, 2007), has been used to structure the presentation of content. Statistics is presented as an investigative process that involves four components: (i) clarifying the problem at hand and formulating questions that can be answered with data; (ii) designing and employing a plan to collect appropriate data; (iii) selecting appropriate graphical or numerical methods to analyze the data; and (iv) interpreting the results. In order to help teachers go beyond procedural memorization and acquire a well-organized body of knowledge, the course emphasizes and revisits a set of core statistical ideas. Through their participation in authentic educational activities such as projects, experiments, computer explorations with real and simulated data, group work and discussions, participating teachers learn where and how the "big ideas" of statistics apply, and develop a variety of methodologies and resources for their effective instruction.

In Modules 4-6 (Weeks 7-13), the focus shifts to classroom implementation issues. Teachers customize and expand upon provided materials, and apply them in their own classrooms with the support of the design team. Once the teaching experiment is completed, they report on their experiences to the other teachers in their group, and also provide video-recorded teaching episodes and samples of their students' work for group reflection and evaluation.

Each module involves a range of activities, readings and contributions to discussion, as well as the completion of group and/or individual assignments. Both the dialogue and the assignments are structured so as to explicitly establish links between theory and practice. Reflective questions create situations for the participating teachers to critically examine the subject matter and to make new connections between theory and their personal and professional experiences. The Marijuana Survey task presented in Figure 1, taken from Watson (2010), is indicative of the activities in which teachers engage during the course.

Look carefully at this task:

Decriminalise drug use: poll

SOME 96 percent of callers to youth radio station Triple J have said marijuana use should be decriminalised in Australia.

The phone-in listener poll, which closed yesterday, showed 9924 - out of the 10,000-plus callers - favoured decriminalisation, the station said.

Only 389 believed possession of the drug should remain a criminal offence.

Many callers stressed they did not smoke marijuana but still believed in decriminalising its use, a Triple J statement said.

Is the sample reported here a reliable way of finding out public support for the decriminalisation of marijuana? Why or why not?

1. What are the big statistical ideas in this problem?
2. Please can you give an example of an appropriate response and an inappropriate response that your students might give?
3. What opportunities would this problem provide for your teaching?
4. A student gave this answer: *"Yes, because 10000 people is enough to get an accurate average of the view of the public"*. How would you move this student's understanding forward?
5. A student gave this answer: *"No, because it is not everyone in Australia voting"*. How would you move this student's understanding forward?
6. A student gave this answer: *"No, because some people could be lying"*. How would you move this student's understanding forward?

Figure 1: The Marijuana Survey task (Watson, 2010)

The course activities encourage critical reflection on workplace practice and productive interaction among course participants. Members of the *EarlyStatistics* consortium with expertise in statistics education act as facilitators of a deeper learning experience by guiding discussions, encouraging the full, thoughtful involvement of all participants and providing feedback.

Media and technology choices

The *EarlyStatistics* pilot course is delivered using a blended-learning method. At the beginning of the course, there is a face-to-face meeting with all participants. Teachers from all over Europe gather together to attend a one-week-long intensive seminar (they can finance their expenses by applying for an in-service training grant). They are first introduced to the objectives and pedagogical framework underpinning the course. They then become familiarized with the facilities offered by the e-learning environment and, more importantly, they get the chance to meet and interact with one another.

The remainder of the course is delivered online, through text, illustrations, animations, audio/video and technology-rich interactive problem-solving activities. The instructional content and services of the project's dedicated information base are used for teaching, support and coordination purposes. In addition to the course content, the site (http://www.earlystatistics.net/) offers access to various other links and resources:

- *Technologically enhanced instructional materials* for statistics teaching and learning.
- *A digital Video Case Library* containing segments of real teaching episodes, obtained from the classrooms of the teachers participating in the pilot delivery.
- *A database of Student Work Samples* developed through contributions made by the participating teachers.
- *Collaboration tools* for professional dialogue and support, including email, conferencing, chat rooms, discussion forums, wikis, etc.
- *Archived forum discussions*.
- *Reports and articles* arising from the project.
- *Links to statistics education resources* available on the Internet.
- *Multilingual interfaces* (English, Greek and Spanish) to partly overcome language barriers.

In order to offer teachers flexibility and to accommodate different time zones, the largest portion of the course is delivered asynchronously. There is also some synchronous communication through the use of technologies such as audio/video streaming and videoconferencing.

Central to the course design is the functional integration of technology and core curricular ideas, and specifically the integration of statistics educational software (the dynamic software Tinkerplots© and Fathom©) and a variety of online activities and resources (e.g., simulations, animations, video clips, etc.). The aim of the latter is to stimulate and engage teachers while providing them with the opportunity to model and investigate real-world statistics-related problems.

Evaluation of *EarlyStatistics*

In *EarlyStatistics*, evaluation was an integral part of the project design. It was a process carried out at every stage of project development in order to ensure that all key activities were performed on time and effectively, and that any necessary revisions or improvements to the project's methodologies, products and outcomes were identified in a timely manner. It included both formative and summative assessment tools, protocols and services, and was conducted both internally and externally. The main external evaluation took place during the pilot delivery of the course and the follow-up classroom experimentation. Multiple forms of assessment were used to collect and document evidence of changes in teachers' pedagogical and content knowledge of statistics, in their attitudes towards the subject and in their teaching practices as a result of participating in the course: pre- and post-questionnaires, video-recording of classroom episodes, teacher and student interviews, samples of student work and use of statistics automatically generated by the online information base.

The overall feedback from the target user groups from all partner countries participating in the pilot delivery of the *EarlyStatistics* course, as well as from external experts in statistics education regarding the course content, services and didactical approaches was generally very positive. Key conclusions drawn from the analysis of user feedback were that *EarlyStatistics* was quite successful at helping teachers improve their pedagogical and content knowledge of statistics by offering interactive, technology-rich instructional materials and services that enhance the teaching and learning process, and also by providing course participants with the opportunity to collaborate with other teachers and thus initiate the construction of a community of practice. Moreover, data obtained from the teaching experimentations in the course participants' classrooms suggest positive gains in student learning outcomes and attitudes towards statistics (for more details see Chadjipadelis, 2008).

In the survey administered on completion of the pilot delivery of the course and the follow-up interviews, teachers were asked to indicate "what they liked the most about the *EarlyStatistics* course". The flexibility and convenience associated with distance education was an aspect of the course appreciated by all 14 course participants. They all considered the distance training nature of *EarlyStatistics* to be an advantage of the course, since it made it possible for them to determine their own place, pace and time of study: "It is a form of training that does not place stifling limits and restrictions of freedom on the teacher"; "You decide your own workload"; "You can follow your own pace of work". Further, a few teachers noted that the distance option gave them the opportunity to attend a course in statistics education offered by experts in the field originating from different European countries.

The promotion of communication and collaboration among teachers was an aspect of the *EarlyStatistics* course that was also considered by all of the course participants to be an important strength of the program. Teachers enjoyed the interaction and the sharing of experiences and ideas with the other teachers: "I liked the interaction with the other teachers. It is useful to share your ideas and problems with other teachers from different educational levels". In particular, teachers praised the fact that *EarlyStatistics* had allowed them, through computer-mediated communication, to share content, ideas and instructional strategies with teachers from different countries and educational systems: "It is good to 'hear' colleagues from other countries that face similar problems like you and sometimes, because of a different view on a point, suggest ideas you didn't think of".

Another aspect of the *EarlyStatistics* course that was also much appreciated by teachers is the fact that the course dialogue and assignments were carefully designed to be learner-centered, and to make explicit links between theory and practice by utilizing participating teachers' own experiences as learning resources. Several of the course participants pointed out that *EarlyStatistics* offered them professional development that addressed their workplace educational needs because it was deeply contextualized in their professional activity: "It is a form of training that respects teachers' professional experience and contributes to the improvement of their educational work through the enrichment of experiences and the exchange of opinions with other teachers that work in different cultural and educational environments."

The *EarlyStatistics* project won, ex-aequo with *Maths4Stats* (a joint project coordinated by Statistics South Africa), the 2009 Best Cooperative Project Award in Statistical Literacy. This prestigious award is given every two years by the International Association of Statistics Education (IASE) "in recognition of outstanding, innovative and influential statistical literacy projects that affect a broad segment of the general public".

Despite the overall success of the pilot course, a number of shortcomings have also been identified. The biggest difficulty experienced by the consortium was in achieving the successful building of an online community of teaching practitioners, which was one of the main objectives of *EarlyStatistics*. From the outset of the project, we were well aware of the challenges in developing such a community, of the fact that merely forming a discussion group and providing the technology does not automatically lead to the establishment of relations and group cohesion (Gordon, 2007). The experience gained from pilot testing the course further alerted us to the fact that community building, particularly in a cross-national context, is very difficult. Despite the fact that we employed several strategies to promote teacher dialogue and collaboration, we experienced similar disappointment to that of Gould (2005), with a lower than expected level of online interaction among participating teachers (Meletiou-Mavrotheris, 2011).

While at the beginning of the course there was considerable enthusiasm and very high participation in the discussion forums, interaction dropped off over time. A total of 229 messages were sent to *EarlyStatistics* over the 13 weeks that the course lasted (76 messages/month on average). However, the vast majority of the messages (167 messages, 73% of all messages sent) were sent in the first six weeks of the course. In contrast to the vibrant interaction and rich dialog characterizing the earlier part of the course, towards the end of the course it was often the case that only 3-4 teachers would actively participate in the discussion forums, while the rest would make minimal or no contributions.

The analysis of the data obtained from the pilot delivery of the *EarlyStatistics* course and follow-up classroom experimentation has provided the consortium with invaluable insights regarding the course's effectiveness in delivering its stated aims. In particular, findings from the pilot delivery have allowed us to identify a number of factors that adversely affected the online participation of course participants (Meletiou-Mavrotheris, 2011). These factors informed the revision of the course to better support community building among participating teachers.

A main factor contributing to our limited success in building an online community of practice during the pilot delivery was the fact that there was no face-to-face meeting with all course participants. There were a few face-to-face meetings with local teachers, but not with the group as a whole. Course participants got the chance to virtually meet teachers from other countries through videoconferencing, but this cannot be considered as effective as face-to-face interaction. As a result, while teachers built strong local groups, their interaction with participants from other countries was limited. In current offerings of the course, teachers are recruited from across Europe, and at the start of the course there is a face-to-face meeting with all participants. This initial in-person meeting reinforces the online engagement of teachers by helping to mitigate the problem of trust and social presence online.

4. Conclusion

In a world where the ability to analyze, interpret and communicate information from data are skills needed for daily life and effective citizenship, developing a statistically literate society has become a key factor in achieving the objective of an educated citizenry. Recognizing teachers' ongoing professional development and learning as a linchpin of instructional innovation and success for their students (Ginsberg, 2003), *EarlyStatistics* has exploited the affordances offered by open- and distance-learning technologies to help improve the quality of statistics instruction in European schools. The project consortium has incorporated into the course design best pedagogical practices in statistics education, adult education and distance learning. The course is based on current pedagogical methodologies utilizing collaboration, statistical investigation and exploration with online interactive problem-solving activities. Particular care has been taken to build on participating teachers' knowledge and experiences, and to promote collaborative and participatory learning. Teachers from different countries have the opportunity to improve their content and pedagogical knowledge of statistics through open-ended investigations, simulations, visualizations, collaboration and reflection on their own and on others' ideas and experience.

The *EarlyStatistics* project outputs and services are useful not only to teachers, but also to academic experts in statistics education, to teacher training institutions and to designers of online professional development programs across Europe and internationally. Academic experts and material developers can become more sensitized to the needs of statistics teachers in different countries, supporting the development of new professional development methodologies and materials grounded on a community-building model. Teacher training institutions can gain a clearer understanding of the issues facing statistics teaching and learning, and can use the project outputs for further improvement of their teacher training programs.

A particularly important issue in the online professional development of teachers is ensuring the successful building of an online community of practice. The first experiences with *EarlyStatistics* concur with the research literature, indicating that the successful building of an online community of practice is very challenging. As Gould and Peck (2005) have pointed out, leading a discussion of substance on a discussion board is more challenging than in a real classroom. For Kling (2003), the transformation of a group into a community is "a major accomplishment requiring special processes and practices" (p. 221). An online community of practice will not automatically take shape through the availability of an online space. Rather, it requires carefully crafted designs – both technical and social (Rourke, 2007).

Teaching online courses is a new, unexplored territory for most statistics instructors. Online instruction is similar yet different from face-to-face learning, and requires new teaching skills and strategies. Online instructors' new role as course facilitators turns them into both guides and learners (Heuer, 2004). In order to facilitate student success and to foster online participation, they must be trained in this new mode of instruction while developing the art of becoming online guides. Online courses should also be subject to continuous evaluation and enhancement. Garfield (2009), whose distance teacher training course has been quite successful in achieving learner participation and collaboration, explain that their online courses are subject to an ongoing cycle of evaluation and improvement. Each time an online course is taught, changes are made to the way in which discussion assignments are structured and used, based on feedback received from students and on careful examination of the patterns of interaction occurring within different discussion groups. *EarlyStatistics* has also adopted an iterative model of continuous improvement. Evaluation continues to play a pivotal role in each subsequent offering of the course. This allows us to continuously improve the quality and effectiveness of *EarlyStatistics*, which is the first online professional development course in the area of statistics education at the European level.

References

CADY, J.: REARDEN, K. (2009). "Delivering online professional development in mathematics to rural educators". *Journal of Technology and Teacher Education*. Vol. 17, pages 281-298.

CHADJIPADELIS, T.; ANDREADIS, I. (2008). *EarlyStatistics Evaluation Report* [internal document]. Project: 226573-CP-1-2005.

DEDE, C. [et al.]. (2006). *Research Agenda for Online Teacher Professional Development*. Cambridge, MA: Harvard Graduate School of Education.

DUTTON, J.; DUTTON, M. (2005). "Characteristics and performance of students in an online section of business statistics". *Journal of Statistics Education*. Vol. 13, No 3.

ESPINEL, C.; BRUNO, A.; PLASENCIA, I. (2008). "Statistical graphs in the training of teachers". In: C. Batanero; G. Burrill; R. Reading; A. Rossman. (2008). *Proceedings of the Joint ICMI/IASE Study Teaching Statistics in School Mathematics. Challenges for Teaching and Teacher Education*. [CD-ROM]. Monterrey, Mexico: ICMI & IASE.

EVANS, S. R. [et al.]. (2007). "Evaluation of Distance Learning in an Introduction to Biostatistics Course". *Statistical Education Research Journal*. Vol. 6, No 2, pages 59-77.

EVERSON, M. G.; GARFIELD, J. (2008). "An innovative approach to teaching online statistics courses" [online document]. *Technology Innovations in Statistics Education*. Vol. 2, No 1. [Accessed: 30 July 2011]. <http://repositories.cdlib.org/uclastat/cts/tise/>

FRANKLIN, C. A. [et al.]. (2007). *Guidelines for Assessment and Instruction in Statistics Education (GAISE) report: A pre-K-12 Curriculum Framework*. Alexandria, VA: American Statistical Association.

GINSBERG, M. B. (2003). *Motivation matters: A workbook for school change.* San Francisco: Jossey-Bass.

GORDON, S.; PETOCZ, P.; REID, A. (2007). "Tools, artefacts, resources and pedagogy – stories of international statistics educators" [online document]. In: P. L. Jeffery (comp.). *Australian Association for Research in Education 2006 Conference Papers*. AARE. [Accessed: 30 July 2011]. <http://www.aare.edu.au/06pap/gor06358.pdf>

GOULD, R.; PECK, R. (2005). "Inspiring Secondary Statistics" [online document]. *MSOR Connections*. Vol. 5, No 3. [Accessed: 30 July 2011]. <http://mathstore.ac.uk/headocs/53inspiringstats.pdf>

HEUER, B. P.; KING, K. P. (2004). "Leading the Band: The Role of the Instructor in Online Learning for Educators" [online document]. *The Journal of Interactive Online Learning*. Vol. 3, No 1. [Accessed: 30 July 2011]. <http://www.ncolr.org/jiol/issues/PDF/3.1.5.pdf>

JOUBERT, M.; SURTHERLAND, R. (2009). *A perspective on the literature: CPD for teachers of mathematics.* University of Bristol: National Centre for Excellence in Teaching Mathematics.

KLING, R.; COURTRIGHT, C. (2003). "Group Behavior and Learning in Electronic Forums: A Sociotechnical Approach". *Information Society*. Vol. 19, pages 221-235.

MCGRAW, R.; LYNCH, K.; KOC, Y. (2007). "The multimedia case as a tool for professional development: An analysis of online and face-to-face interaction among mathematics pre-service teachers, in-service teachers, mathematicians, and mathematics teacher educators". *Journal of Mathematics Teacher Education*. Vol. 10, No 2, pages 95-121.

MELETIOU-MAVROTHERIS, M. (2011). "Online Communities of Practice as Vehicles for Teacher Professional Development". In: A. Juan; M. A. Huertas; S. Trenholm; C. Steegmann (eds). *Teaching Mathematics Online: Emergent Technologies and Methodologies.* IGI Global. Pages 142-166.

MELETIOU-MAVROTHERIS, M.; SERRADÓ, A. (2011). "Distance Education of Statistics Teachers". In: C. Batanero; G. Burrill; C. Reading (eds). *Teaching Statistics in School Mathematics-Challenges for Teaching and Teacher Education: A Joint ICMI/IASE Study: The 18th ICMI Study*. DOI 10.1007/978-94-007-1131-0_36. Springer Dordrecht Heidelberg London New York. Pages 383-394.

RIVERIN S.; STACEY, E. (2007). "The Evolution of an Online Community – A Case Study". *Research and Practice in Technology Enhanced Learning.* Vol. 2, No 3, pages 267-297.

ROURKE, L.; KANUKA, H. (2007). "Barriers to Online Critical Discourse". *International Journal of Computer-Supported Collaborative Learning.* Vol. 2, No 1, pages 105-126.

TUDOR, G. (2006). "Teaching Introductory Statistics Online – Satisfying the Students". *Journal of Statistics Education.* Vol. 14, No 1.

VERMEIRE, L.; CARBONEZ, A.; DARIUS, P.; FRESEN, J. (2002). "Just-in-time Network Based Statistical Learning: Tools Development and Implementation". In: B. Phillips (ed.), *Proceedings of the Sixth International Conference on Teaching Statistics (ICOTS6)*. Cape Town, South Africa: IASE.

WATSON, J. M.; NATHAN, E. L. (2010). "Biased Sampling and PCK: The Case of the Marijuana Problem". In: L. Sparrow; B. Kissane; C. Hurst (eds). *Shaping the future of mathematics education: Proceedings of the 33rd Annual Conference of the Mathematics Education Research Group of Australasia*. Fremantle: MERGA.

ZHAO, Y.; ROP, S. (2001). *A critical review of the literature on electronic networks as reflective discourse communities for inservice teachers* [online document]. [Accessed: 30 July 2011]. <http://www.ciera.org/library/reports/inquiry-3/3-014/3-014.pdf>

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ARTICLE

On How Moodle Quizzes Can Contribute to the Formative e-Assessment of First-Year Engineering Students in Mathematics Courses

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Abstract
Given the importance of formative assessment in the context of the European Higher Education Area, it is necessary to explore new tools to implement innovative strategies for the formative assessment of students. Moodle's quiz module represents an alternative to traditional tools, such as paper-and-pencil tests. In 2008, we carried out a project subsidised by the Institute of Education Sciences at the Universitat Politècnica de Catalunya - BarcelonaTech (UPC), the main aim of which was to elaborate a number of Moodle question pools and to design, implement and assess a series of quizzes from these pools. The project covered the compulsory undergraduate subjects in applied mathematics included in the first- and second-year syllabuses for all branches of Engineering. From the students' results, it was then necessary to examine and revise the reliability of the quizzes as an assessment tool of the teaching and learning process. The analysis of the psychometric coefficients provided by Moodle proved to be a useful tool for assessing whether the questions had an appropriate level of difficulty and were suitable for discriminating between good and bad performers. Taking into account the psychometric analysis of this first project, in 2009 we initiated a new project, in which we planned to revise thoroughly the quizzes created in the former project, to improve their suitability as an assessment tool. This paper shows: i) the students' results in the quizzes performed in the two academic years in the courses Mathematics 1 and Mathematics 2 – both taught in the first year of the four bachelor's degree programmes in Biological Systems Engineering organised by the School of Agricultural Engineering of Barcelona at the UPC, as well as the students' attitudes towards activities of this kind; and ii) the revision and fine-tuning of the quizzes from the psychometric analysis to improve their reliability. Finally, the analysis of the results reported leads to a discussion on the advisability of using this tool for the formative assessment of students.

Keywords
mathematics, quizzes, Moodle, assessment, psychometric analysis

Los cuestionarios del entorno Moodle: su contribución a la evaluación virtual formativa de los alumnos de matemáticas de primer año de las titulaciones de Ingeniería

Resumen
En el contexto del Espacio Europeo de Educación Superior, en el que la evaluación formativa desempeña un papel esencial, es necesario explorar nuevas herramientas con el fin de implementar estrategias innovadoras de seguimiento y evaluación de los estudiantes. El módulo de cuestionarios en el entorno Moodle representa una alternativa frente a las metodologías tradicionales, como pueden ser las pruebas escritas. En el marco de las ayudas para la mejora de la docencia concedidas por el Instituto de Ciencias de la Educación de la Universidad Politécnica de Cataluña - BarcelonaTech (UPC), durante el curso académico 2008/2009 se llevó a cabo un proyecto docente cuyo objetivo principal era el diseño de preguntas tipo test y su posterior implementación en cuestionarios del entorno Moodle para asignaturas de matemáticas y estadística correspondientes a primeros cursos de titulaciones de ingeniería. Con los resultados de los cuestionarios realizados por los estudiantes, se vio que era conveniente analizar y revisar su fiabilidad y adecuación para avalar estas actividades de evaluación del proceso de aprendizaje. El análisis de los coeficientes psicométricos facilitados por Moodle resultó ser una herramienta útil a la hora de valorar si las cuestiones propuestas tenían el nivel de dificultad adecuado y si, en consecuencia, eran convenientes para discriminar entre buenas y malas prácticas. En el marco de otro proyecto, también subvencionado por la UPC, durante el siguiente curso académico 2009/2010 se revisaron de forma exhaustiva los cuestionarios implementados con el fin de mejorar su eficiencia como herramienta de evaluación. En este trabajo se presentan: i) los resultados de los cuestionarios realizados por los estudiantes durante esos dos cursos académicos en las

asignaturas Matemáticas 1 y Matemáticas 2 de primer año de los cuatro grados de Ingeniería de Biosistemas de la Escuela Superior de Agricultura de Barcelona de la UPC, así como la opinión de los estudiantes sobre este tipo de actividad, ii) la revisión y adaptación de los cuestionarios a partir de los índices psicométricos para mejorar su eficiencia. Finalmente, a partir de los resultados analizados se hace una reflexión sobre la conveniencia de utilizar este tipo de herramientas para la evaluación formativa de los estudiantes.

Palabras clave
matemáticas, cuestionarios, Moodle, evaluación, análisis psicométrico

1. Introduction

The Bologna Declaration and the implementation in 2010 of the European Higher Education Area (EHEA) brought about crucial changes both in the curriculum and in teaching-learning methodologies in university studies (ENQA, 2005). The EHEA promotes a student-centred system based on the student workload required to achieve the objectives of a programme of study. These objectives should be articulated in terms of learning outcomes to be acquired. Learning outcomes are sets of competencies, expressing what the student will know, understand or be able to do after completion of a process of learning. Competencies represent a dynamic combination of attributes, abilities and attitudes, which should correspond to specified learning outcomes. In this framework, student workload consists of the time required to complete all planned learning activities such as attending lectures, seminars, independent and private study, project preparation and examinations. The assessment of students is a cornerstone of the EHEA and is expected to "measure the achievement of the intended learning outcomes and other programme objectives" (ENQA, 2005). ENQA's guidelines for the assessment of students also include desirable procedures that should be followed in the assessment process.

According to the EHEA guidelines, it is clear that formative practices are a central component in the assessment of students. Among the aspects that lecturers must take into account when designing and developing tools for formative assessment of the teaching-learning process, we would stress the following: i) To reflect on actions before, during and after the learning process, on the part of the lecturer and the learner; ii) To include assessments for both learning outcomes and learning processes; iii) To provide feedback to improve both the teaching and the learning processes; iv) To incorporate student self-assessment and self-regulation procedures; and v) To explain and share evaluation criteria with students.

Furthermore, several studies have pointed out the increasing role of information and communication technologies (ICTs) in the field of assessment (Delgado and Oliver, 2006; Graff, 2004; Steegmann et al., 2008), to the extent that *e-assessment* has emerged as a new issue in the educational arena (Brinck and Lautenbach, 2011; Crews and Curtis, 2011; Daly et al., 2010; Ferrão, 2010). Given the importance of formative assessment in the context of the EHEA, it is fundamental to explore new evaluation strategies to innovate assessment methods. As Ferrão (2010) points out, the system of e-assessment must have the hardware and software necessary for test generation and administration.

Most Spanish universities have adopted Moodle as a learning management system (LMS) to help educators create quality online courses and administer learner outcomes (Steegmann et al. 2008). Therefore, in this paper we focus on the quiz module provided by Moodle. This module allows for the creation of quizzes with different question types, adapted to the specific objectives to be achieved at any step in the teaching-learning process, supplying prompt, automatic feedback. A powerful tool for monitoring and diagnosing students' learning, Moodle's quiz module represents an alternative to traditional face-to-face courses and paper-based testing. Regarding the quality of the e-assessment system, Moodle's quiz module supplies statistical methods to measure the reliability of the tests (Ferrão, 2010). It has been argued that, in relation to the use of ICTs, the boundaries between formative and summative assessment become blurred (Daly et al., 2010). However, if technologies are used to carry out low-stakes assessment activities on a regular basis, they can contribute to formative assessment. Moodle quizzes not only prove suitable for carrying out such activities, but they can also be modified and adapted according to learners' needs. As discussed in Daly et al. (2010), adaptivity is a key feature of e-assessment, since feedback is used formatively by learners to adapt their conceptions and approaches to a task, and by lecturers to adapt a task to learners' needs. We are well aware that quizzes have become a widely used tool for assessment in recent years (Ferrão, 2010). However, to our knowledge, there is no detailed survey on how to make the most of psychometric coefficients to refine quizzes implemented in undergraduate mathematical courses.

This paper reports on the main outcomes of two educational projects where Moodle quizzes were used as a tool for formative e-assessment in the context of two compulsory undergraduate mathematical courses. The projects aimed:

1. To design a number of quizzes to assess regularly the topics of the two courses, with a subsequent analysis of the learners' results and their correlation with other teaching-learning activities involved in the courses, as well as to collect the students' attitudes towards e-assessment.
2. To carry out a psychometric analysis as a means of feedback on the learning activities in order to adapt them to the learners' needs and therefore to refine and improve their reliability as a tool for formative e-assessment.

2. Material and methods

Since 2009, the School of Agricultural Engineering of Barcelona (ESAB) at the Universitat Politècnica de Catalunya - BarcelonaTech (UPC) has offered the following bachelor's degree qualifications in Biosystems Engineering: Degree in Agricultural Engineering, Degree in Biological Systems Engineering, Degree in Agro-Environmental and Landscape Engineering, and Degree in Food Engineering. The four bachelor's degrees share a set of compulsory subjects in the first and second years, which count as six European Credit Transfer System (ECTS) credits each. Two first-year courses in mathematics,

Mathematics 1 and Mathematics 2, are included in this initial common set. It is worth noting here that the essentially biological profile of the ESAB has arguably contributed to the students' poor motivation in mathematical and statistical areas, and this has traditionally resulted in low pass rates. In order to improve the learning outcomes and to motivate the students, we decided to launch a series of low-stakes tasks as an incentive (Lim et al., 2011). Yet, if we wanted to meet the EHEA guidelines on assessment while dealing with a growing number of students, this would doubtless mean an increase in teaching staff workload. In order to carry out continuous assessment of our students without investing an excessive amount of time marking, it seemed appropriate to resort to the range of e-tools available.

In 2005, the UPC started to use Moodle, an open-source LMS that offers a wide variety of teaching tools (Cole, 2005). In order to make the most of the tools available, we started exploring Moodle's assessment facilities. In 2008/2009, we carried out a project subsidised by the Institute of Education Sciences at the UPC, the main aim of which was to design, elaborate and implement a substantial range of Moodle question pools for quizzes ("*Creació de qüestionaris des de l'entorn Moodle per a assignatures de matemàtiques i estadística corresponents a primers cursos de titulacions d'enginyeria*"). The project covered the compulsory undergraduate subjects in applied mathematics included in the first- and second-year syllabuses for all branches of Engineering. In practice, it was mainly centred on Mathematics 1 (M1) and Mathematics 2 (M2), compulsory for all students enrolled in the ESAB. In this project, we analyzed the students' answers, and carried out a psychometric analysis to identify the appropriateness of the questions asked in the quizzes. It is important to stress that a preliminary experience was carried out with a small group of students the year before the new bachelor's degree system started. This initial experience seemed to suggest that Moodle quizzes were certainly useful for the promotion of student involvement in mathematical subjects.

However, it is essential to bear in mind that the whole process should be permanently revised and updated. Therefore, carrying out an evaluation of the various experiences in Mathematics 1 and Mathematics 2 provided the research group with insights into the entire assessment process.

From those initial experiences, we planned to generate improved quizzes suitable for the mathematics courses mentioned above. The psychometric analysis provided by Moodle was a great tool for assessing whether the questions were suitable for discriminating between good and bad performers, with an appropriate level of difficulty.

Taking account of the psychometric analysis of that first project, in 2009/2010 we carried out a new project in which we planned to revise thoroughly the quizzes created in the former project in order to improve their reliability as an assessment tool ("*Revisió i millora de l'eficiència de qüestionaris MOODLE implementats en assignatures de matemàtiques i estadística corresponents a primers cursos de titulacions d'enginyeria*").

In order to supervise the students' progress at different stages of the learning process (Heck and Van Gastel, 2006), we created quizzes for different contexts, such as diagnostic and post-performance tests, computer lab sessions and chapter checking after the accomplishment of each unit of content. This contribution focuses on the set of Moodle quizzes that were designed as take-home assignments for chapter checking, to be completed within a given time frame. The topics covered by each of the quizzes in Mathematics 1 and Mathematics 2 were aligned with the learning goals and required outcomes of the course (Tables 1 and 2). Since different kinds of questions can help to develop different skills (Smith et al., 1996; Blanco et al., 2009), the questions used in these quizzes were of several types: multiple-choice, true/false, short-answer, numerical, matching and embedded (cloze) (Table 3).

Summative assessment in both courses is carried out on the basis of a weighted formula computed as follows: two or three written tests during the semester (45%); a cumulative final written exam (40%); computer lab sessions (5%); quizzes (5%); and several homework and coursework assignments (5%). It is within this framework that the quizzes have to be considered.

Table 1. Topics covered by quizzes in Mathematics 1.

Q1	Q2	Q3	Q4	Q5	Q6
Functions of a real variable	Functions of several real variables	Determinants and Systems of linear equations	Complex numbers	Optimization of functions of a real variable	Optimization of functions of several real variables

Table 2. Topics covered by quizzes in Mathematics 2 (ODEs: Ordinary Differential Equations).

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
Basic integration by substitution		Integration by substitution	Integration by parts	Integration by partial fractions	General topics on ODEs	Separable ODEs	Homogeneous ODEs

Table 3. Number of questions and question types corresponding to the first project (changes made in the second project in brackets).

	Number of questions	Multiple-choice	True/False	Matching	Short-answer/ Numerical	Cloze
M1	83	65 (60)	10 (18)	3	5 (2)	
M2	59	19	38			2

3. Results and discussion

As an interactive and dynamic tool, Moodle quizzes have an impact on the lecturers' and students' attitudes towards computer-assisted assessment. Moreover, the fact that the number of enrolled students has been growing in recent years means that we have to apply the EHEA guidelines (ENQA 2005) to groups of 60 students or more. Hence, the automatic assessment offered by the quizzes can free up time for lecturers to concentrate on other aspects of the learning process (Blanco et al., 2009). As mentioned earlier, the UPC's LMS – Atenea – is based on Moodle. From the very beginning, the university's strategy has been to encourage lecturers and students to use this LMS to work towards the learning and teaching process as understood by the EHEA. So, the use of Moodle's quiz module, as described in this paper, is a move in that direction. This section is organized as follows. The first subsection analyzes the students' results for the quizzes set in both academic years (2009/2010 and 2010/2011). The second subsection presents the main results for the psychometric analysis of the quizzes. The third and final subsection discusses the students' opinions on the quizzes.

3.1. Analysis of the students' results

In the context of our projects, Moodle's quiz module provided information about which questions our students got wrong or partially right, overall quiz results and individual responses. In both projects, we performed a linear regression analysis relating the score mean of the quizzes to the final mark of Mathematics 1 and Mathematics 2, computed using the above-mentioned weighted formula (Figure 1). Overall, the analysis was significant and displayed a good positive linear correlation, with the following correlation coefficients: 0.69 (p-value<0.001) for M1 in 2009/2010 (with N_1=91 students); 0.55 (p-value<0.001) for M2 in 2009/2010 (with N_2=78 students); 0.44 (p-value<0.001) for M1 in 2010/2011 (with N_3=176 students); and 0.67 (p-value<0.001) for M2 in 2010/2011 (with N_4=154 students). These results led us to conclude that Moodle quizzes can be regarded as a suitable tool to inform students of their performance throughout the learning process, in line with Ferrão (2010).

It is interesting to note that, from the data, it is possible to recognize the different strategies that the students use to pass the course. These different behaviours can explain some of the atypical or extreme observations collected over the two years in question. The year 2009/2010 was an exceptional course because there were only new students in the two courses, that is, none of them were retaking the year. In contrast, in the following year, new students and students retaking the course were mixed in the same classroom. The behaviour of the latter was noticeable; their quiz results were different from those of the former (Figure 1). Moreover, it is evident that the results for the Mathematics 2 course were better than those for the Mathematics 1 course, especially in 2009/2010. This is understandable in the following context: i) the nature of the topics of this second subject of mathematics is different from the first one, with new topics for all the students and, in some way, independent from those studied in previous mathematics courses at high school (Tables 1 and 2); ii) the students of Mathematics 2 have already gone through a previous mathematics course and have therefore learnt how to adapt successfully to the environment; and iii) the students who chose to pursue the second subject are the good students from the previous semester (that is to say, they passed Mathematics 1) or, if they were retaking the course, they may have had some advantage over the students taking the subject for the first time. This aspect becomes much more evident in the year 2010/2011, Mathematics 2, as Figure 1 shows. The four scatter plots show a higher concentration of points in the first and third quadrants. When it comes to Mathematics 2 in 2010/2011, it is true that marks are mainly concentrated in the first quadrant only. This means that most of the students who took the quizzes, passed both the quizzes and the course in general, thus providing more evidence supporting the particular nature of Mathematics 2 observed in the second academic year.

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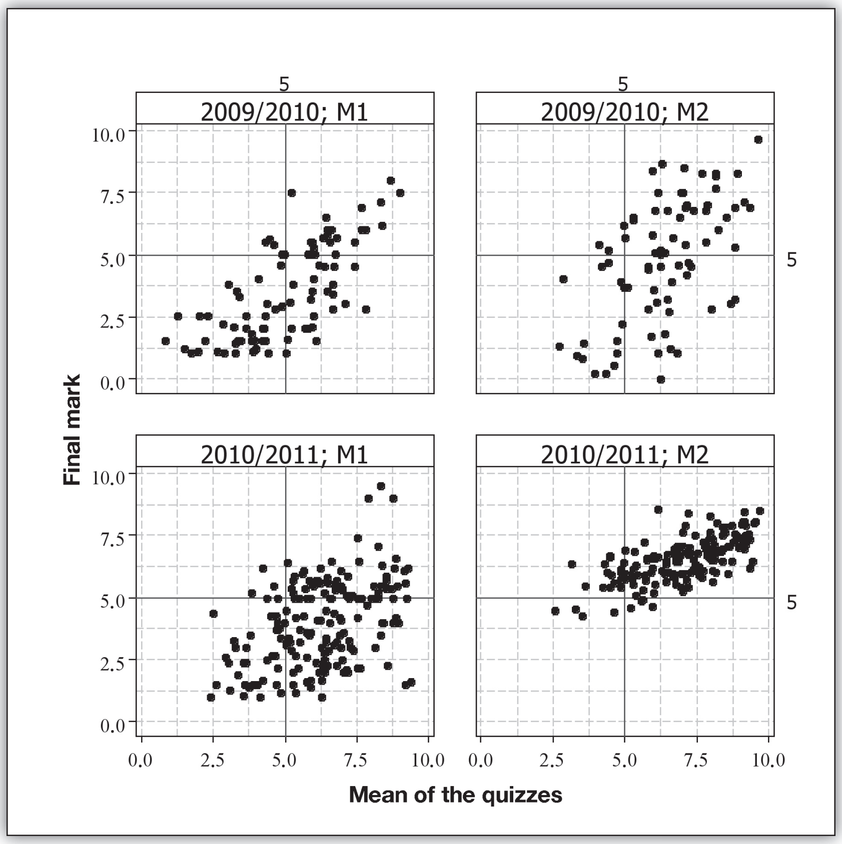

Figure 1. Scatter diagrams of the mean of the students' scores in the quizzes and the final mark in the two subjects (M1: Mathematics 1, and M2: Mathematics 2) in the years 2009/2010 and 2010/2011.

3.2. Psychometric analysis

As Ferrão (2010) argues, the e-assessment system must provide a set of tools to analyze the reliability of the tests and, consequently, to ensure the quality of the system. Psychometric analysis is a great tool for assessing whether the quizzes are a reliable instrument for measuring the students' performance, attitudes and abilities (Heck and Van Gastel, 2006). Moodle's quiz module performs the item analysis of a quiz, a particular tool associated with psychometrics. Having performed the item analysis, the module allows all the statistical reports to be exported as a spreadsheet file, rendering all the information easier to manage.

In this section, we discuss two parameters provided by the item analysis of the quizzes: the Facility Index (FI) and the Discrimination Coefficient (DC). These parameters, calculated as explained by classical test theory, can help us answer whether the questions are well chosen in order to demonstrate concepts and of an appropriate level of difficulty, and whether the questions are suitable enough to discriminate between good and bad performers. The FI describes the overall difficulty of the questions. This index represents the ratio of users that answer a question correctly. In principle, a very high or low FI suggests that a question is not useful as an instrument of measurement. The DC is a correlation coefficient between scores at the item and at the whole quiz level, ranging from -1 to +1. This is another measure of the separating power of the item to distinguish proficient from weak learners.

Although it is necessary to be cautious when relying upon item-discrimination parameters (Burton, 2001), we opted for the DC because it is associated with the Moodle tools available. In addition, since the quizzes did not contain disparate topics, as Tables 1 and 2 show, they met one of the requirements indicated by Burton (2001) for the performance of a more reliable item-discrimination analysis.

At the beginning of the first project, we decided to group the DC values into three categories: Low (DC < 0.33), Medium and High (DC > 0.66). In order to discard those questions with FI values that were either too low or too high, the boundaries were set at 15 and 85, respectively. Quizzes with just a few questions with FI values between 15 and 85 should be newly constructed, as should those with low DC values. In 2009/2010, we set ourselves the goal of revising and redesigning those quizzes with low DC values or with FI values that were either too low or too high.

When it comes to Mathematics 1, from the information provided by Moodle, only those questions with FI values that were either very low or very high should be rewritten, as should those with low DC values. In Blanco and Ginovart (2010b), there is a detailed description of how the revision of questions was tackled. Once revised, the quizzes were run again and a new psychometric analysis was carried out. Table 4 shows that the results of the psychometric analysis obtained in the second year are generally better than those obtained in the first year. Another way to display the results of the psychometric analysis is to focus on the individual quiz questions, rather than on the quizzes as units. The plots in Figure 2 and Figure 3 seem to indicate an improvement in the psychometric analysis after the revision, with higher DC values in the second year than in the first year.

Table 4. Mathematics 1: Psychometric analysis corresponding to 2009/2010 and 2010/2011.

M1		*FI (%)*		*DC*		
		Range	*% of questions with FI between 15 and 85*	*% of questions with Low DC*	*% of questions with Medium DC*	*% of questions with High DC*
Q1	2009/2010	14-82	93.3	20	80	0
	2010/2011	36-84	100	0	100	0
Q2	2009/2010	32-85	100	13	74	13
	2010/2011	41-91	80	7	80	13
Q3	2009/2010	22-87	94.1	18	76	6
	2010/2011	25-96	64.7	12	82	6
Q4	2009/2010	57-86	90	0	90	10
	2010/2011	23-87	90	20	70	10
Q5	2009/2010	24-73	100	21	50	29
	2010/2011	21-86	92.9	7	86	7
Q6	2009/2010	29-66	100	8	76	16
	2010/2011	18-78	100	8	76	16

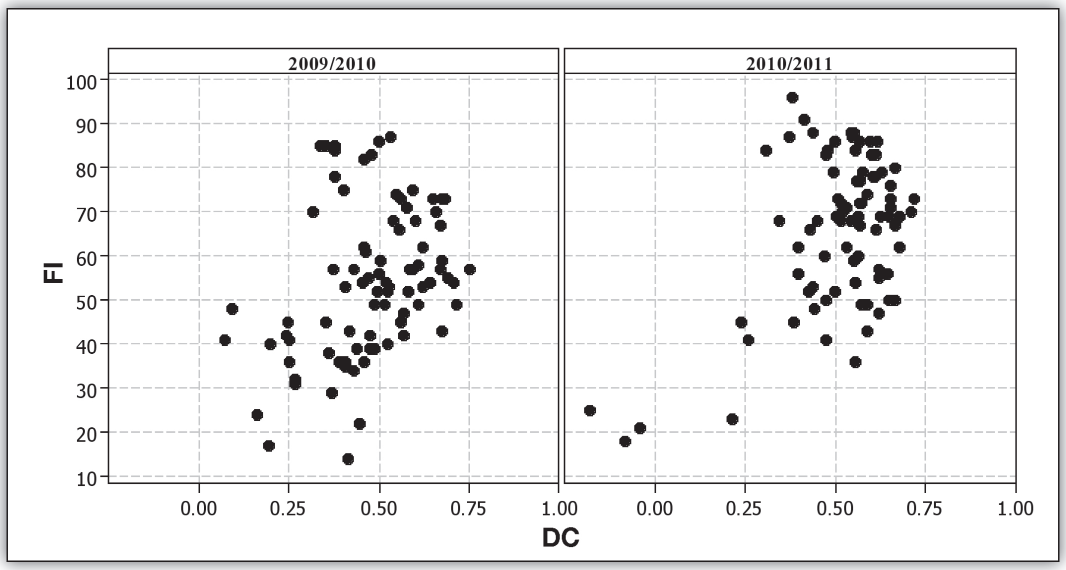

Figure 2. Mathematics 1: Scatter plots of FI and DC, corresponding to all the questions used in the six quizzes in the years 2009/2010 and 2010/2011.

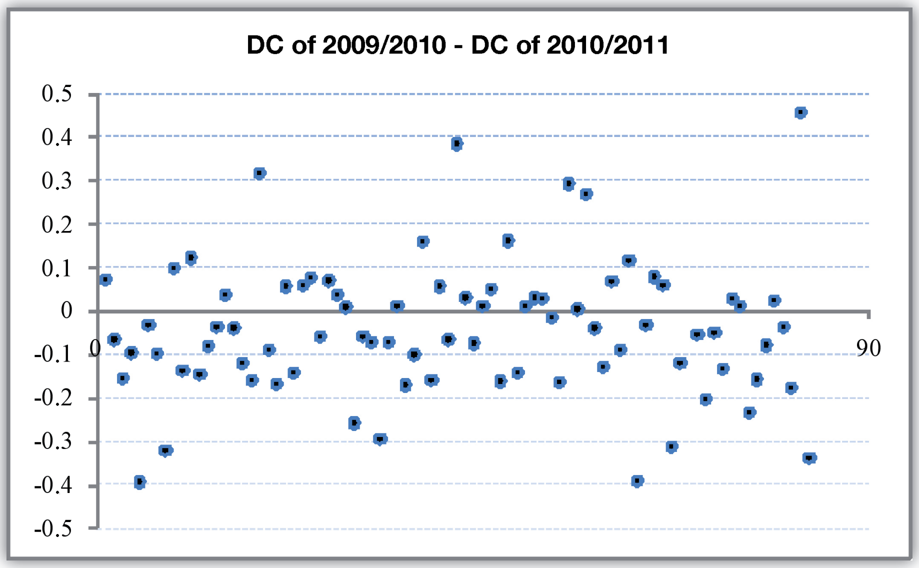

Figure 3. Mathematics 1: Plot of the difference between the DC values in 2009/2010 and in 2010/2011 for each question.

With regard to the eight quizzes performed in Mathematics 2, in the first year (2009/2010), the solutions for the quizzes were made available on the UPC's virtual teaching campus. There was therefore a risk that this, together with the fact that the quizzes were take-home assignments, might tempt the students to copy the answers from the previous year. Consequently, to prevent the students from cheating, in the second year 2010/2011, we redesigned the quizzes already answered the previous year by introducing a few changes, mainly numerical, enough to maintain the essence and reliability of the quizzes. Nevertheless, this could not be taken for granted, since factors alien to the questions, such as changes in the student cohort or changes in the teaching team, might have an impact on the results of the item analysis of a particular quiz. As Figure 4 suggests, such changes in the questions, however slight, could lead to different outcomes, depending on the features of the group of students involved. The positive results obtained in the second year, as noted above, are reflected in higher FI values in general. The random distribution of values around zero in Figure 5 aligns with the fact that no specific action was taken to improve the DC of the quiz questions of Mathematics 2, in contrast to Figure 3, where the values tend to concentrate in the area below zero. Notwithstanding the slight changes made to the quizzes, it is possible to assert that they maintained their DC values in the main. This is therefore acceptable in terms of arguing in favour of the DC's reliability as a psychometric parameter in our study.

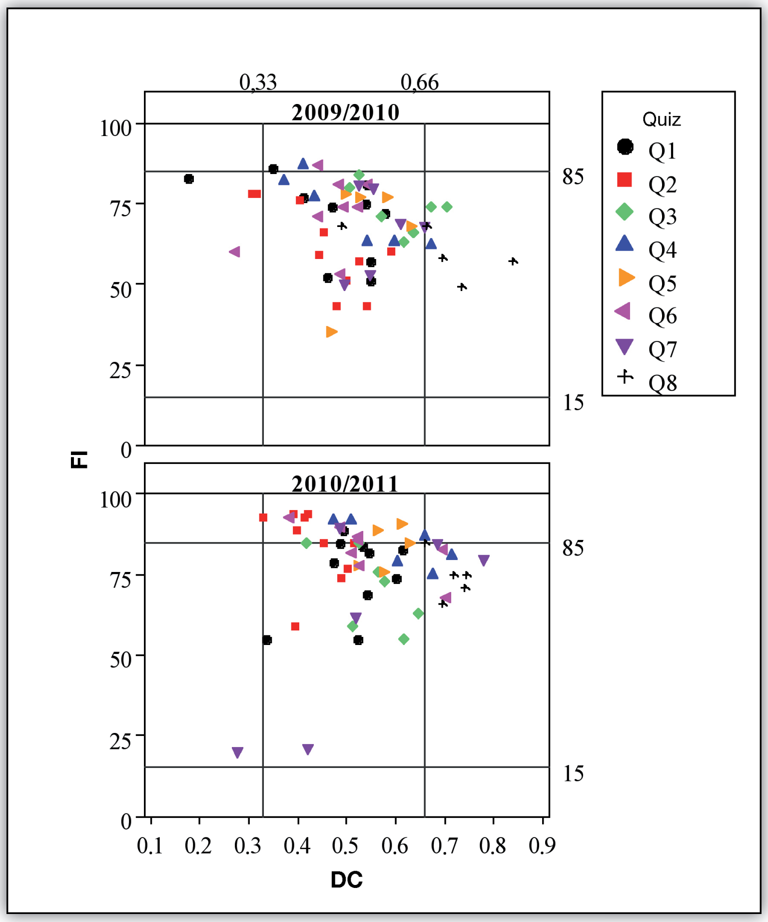

Figure 4. Mathematics 2: Scatter plots of FI and DC, corresponding to all the questions used in the eight quizzes in the years 2009/2010 and 2010/2011.

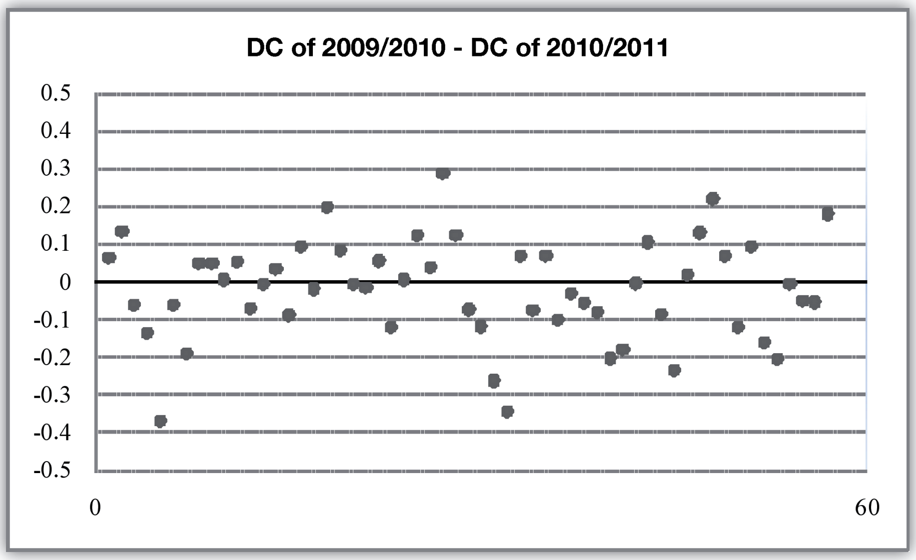

Figure 5. Mathematics 2: Plot of the difference between the DC values in 2009/2010 and in 2010/2011 for each question.

3.3. Analysis of the students' ratings of Moodle quizzes

Some years before the creation of the four bachelor's degree courses in Biosystems Engineering at the UPC, the teaching and learning of mathematical topics at the ESAB was hindered by the students' underachievement, absenteeism and lack of motivation. In order to overcome such obstacles, we decided to work on a new design for the subjects with a substantial increase in the use of computer-assisted methodologies. Therefore, we designed a methodology based on the use of electronic tools aiming at solving standard problems and fostering lecturer-student communication.

At the end of each semester of the academic years 2009/2010 and 2010/2011, we asked our students to rate certain aspects of the quizzes performed and of the use of Moodle. Even though this is not the only source of feedback, the students' ratings provide an excellent guide for designing the teaching process and, in particular, for assessing student motivation. The interaction between the lecturer and the learner helps the former to adapt the learning and assessment tasks to the latter's needs (Daly et al., 2010). Table 5 gives a summary of their answers.

The students were also invited to note down the positive and negative aspects of the quizzes. It is important to underscore the following positive aspects, as expressed by the students themselves:

"It is an easy way to put into practice the theoretical concepts learnt in class"
"Quizzes are entertaining"
"Quizzes provide an instantaneous correction of my responses"
"Quizzes helped me to be in contact with the subject"
"It is an indirect way to enhance the study"

When it comes to the negative aspects, the students not only tended to regard the time available to perform a quiz as insufficient, but they also complained about the poor feedback provided once the quizzes had been performed. Moreover, when asked what improvements they would suggest, the students placed emphasis on getting more feedback on the answers.

Yet, from the results shown in Table 5 and the positive and negative aspects mentioned above, our overall impression is that the students of Mathematics 1 and 2 regarded the quizzes performed positively, both in 2009/2010 and in 2010/2011.

Here it should be noted that the authors undertook a similar project dealing with Moodle quizzes on a Statistics course (Blanco and Ginovart, 2010a). The results of this experience, in keeping with the results obtained in Mathematics 1 and Mathematics 2, would support the advisability of using this type of formative assessment in teaching and learning in higher education.

Table 5. Mathematics 1 and Mathematics 2: The students' ratings.

QUESTION	*2009/2010 M2 n=83 (%)*	*2010/2011 M1 n=158 (%)*
Have you used Moodle before on this course?		
Not at all	21.7	7.0
Rarely	30.1	16.5
Sometimes	32.5	53.8
Often	10.8	17.1
Always	4.8	5.7
Overall, I would rate the quizzes performed as		
Very poor	0.0	3.1
Poor	12.0	8.8
Satisfactory	41.0	40.9
Good	33.7	40.3
Very good	13.3	6.9
The quizzes helped me to understand some of the topics covered in the		
theoretical classes	4.8	6.3
Strongly disagree	15.7	8.2
Disagree	21.7	36.1
Neutral	42.2	41.1
Agree	15.7	8.2
Strongly agree		
Once answered, I got enough information about correct answers		
Strongly disagree	4.8	8.2
Disagree	22.9	22.0
Neutral	31.3	34.6
Agree	30.1	25.8
Strongly agree	10.8	9.4
Performing the quizzes has made me more interested in the subject		
Strongly disagree	2.4	8.2
Disagree	16.9	16.5
Neutral	41.0	52.5
Agree	30.1	17.7
Strongly agree	9.6	5.1

QUESTION	*2009/2010 M2 n=83 (%)*	*2010/2011 M1 n=158 (%)*
I think my scores in quizzes were fair		
Strongly disagree	3.7	2.5
Disagree	4.9	3.8
Neutral	25.6	19.6
Agree	41.5	53.2
Strongly agree	24.4	20.9

4. Conclusions

This paper has presented the results obtained from two projects subsidised by the Institute of Education Sciences at the UPC, the main aim of which was to design and implement a number of Moodle quizzes for the formative e-assessment of students enrolled on mathematics courses for Engineering bachelor's degrees. Subsequently, the reliability of the quizzes as assessment tools was analyzed to ensure the quality of the e-assessment system proposed.

Following the ENQA's report about the standards and guidelines for quality assurance in European higher education, the design and development of the Moodle quizzes involved a reflection that was clearly motivated by the diverse aspects of the teaching-learning process, on the part of the lecturer and the learner.

First of all, it was fundamental to prove whether the consistency of the e-assessment system used aligned with that of the traditional assessment tools used so far. The correlation between scores in the quizzes and the final mark of each subject (Mathematics 1 and Mathematics 2) for the years 2009/2010 and 2010/2011 showed that Moodle quizzes could be regarded as a suitable tool to inform students of their performance throughout the learning process. In addition, the particular use of the quizzes as low-stakes assessment activities for chapter checking contributed to the promotion of student self-regulation and regular work throughout the year. Therefore, this paper provides evidence that Moodle quizzes represent a consistent alternative to open-ended tests in terms of continuous and formative assessment.

In order to meet the requirements of formative assessment, the e-assessment system had to supply tools for the lecturers to adapt an activity to the learners' needs, thus improving its reliability from the feedback obtained. The item analysis provided by Moodle's quiz module turned out to be an interesting psychometric tool to estimate, refine and improve the reliability of quiz questions. In relation to the psychometric analysis performed with the 14 quizzes and with the responses of around 500 students, we achieved a significant step forward in the treatment and comprehension of two indicators, namely, the Facility Index and the Discrimination Coefficient.

Finally, a key aspect in the design and development of the e-assessment system was to check whether the students had a favourable view of it. The fact that the students' ratings of the Moodle quizzes were very positive reinforced the idea that activities of this kind were suitable for mathematics teaching and learning. But not solely mathematics, since the system could be extrapolated naturally to other courses. The results reported in this paper, as well as the students' attitudes, are very encouraging in terms of continuing to work with this e-assessment system and even extending it to other disciplines in the future.

The experience acquired in the development of the reported projects, together with the data generated by the implementation of the quizzes, allowed us to visualize an optimal way to drive forward the effective use of Moodle's quiz module for the formative assessment of students in keeping with the EHEA guidelines. It is worth noting that, by means of this e-assessment system, we managed to carry out the continuous formative assessment of a considerable number of students without overburdening the lecturers with marking or jeopardising assessment quality. This would not have been possible if we had not made full use of Moodle as the LMS supported by our university; this greatly facilitated not only the implementation of tools, but also the collection and analysis of the results. In short, from the results presented in this paper, we can conclude that Moodle quizzes are a consistent and reliable tool for formative e-assessment and consequently we hope that our study will become a reference for further uses of the quiz module.

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References

BLANCO, M.; GINOVART, M. (2010a). "Moodle quizzes for assessing statistical topics in engineering studies". In: K. Resetova (ed.). *Proceedings of the Joint International IGIP-SEFI Annual Conference 2010. Diversity unifies – Diversity in Engineering Education*. Brussels: SEFI.

BLANCO, M.; GINOVART, M. (2010b). "Análisis de la eficiencia de cuestionarios Moodle como herramienta de evaluación de asignaturas de matemáticas correspondientes a primeros cursos de titulaciones de ingeniería". In: *Proceedings of the VI CIDUI: New Areas of Quality in Higher Education. A comparative and trend analysis*. Barcelona: UPC.

BLANCO, M.; ESTELA, M. R.; GINOVART, M.; SAÀ, J. (2009). "Computer Assisted Assessment through Moodle Quizzes for Calculus in an Engineering Undergraduate Course". *Quaderni di Ricerca in Didattica (Scienze Matematiche)*. Vol. 9, No 2, pages 78-84.

BRINCK, R.; LAUTENBACH, G. (2011). "Electronic assessment in higher education". *Educational Studies*. Vol. 37, No 5, pages 503-512.

BURTON, R. F. (2001). "Do Item-discrimination Indices Really Help Us to Improve Our Tests?". *Assessment & Evaluation in Higher Education*. Vol. 26, No 3, pages 213-220.

COLE, J. (2005). *Using Moodle. Teaching with the popular open source course management system*. Sebastopol (CA): O'Reilly Community Press.

CREWS, T. B.; CURTIS, D. F. (2011). "Online Course Evaluations: Faculty Perspective and Strategies for Improved Response Rates". *Assessment & Evaluation in Higher Education*. Vol. 36, No 7, pages 865-878.

DALY, C.; PACHLER, N.; MOR, Y.; MELLAR, H. (2010). "Exploring formative e-assessment: using case stories and design patterns". *Assessment & Evaluation in Higher Education*. Vol. 35, No 5, pages 619-636.

DELGADO, A. M.; OLIVER, R. (2006). "La evaluación continua en un nuevo escenario docente / Continuous assessment in the new teaching scenario". *RUSC*. Vol. 3, No 1, pages 1-13.

ENQA (2005). *ENQA report on Standards and Guidelines for Quality Assurance in the European Higher Education Area*. Helsinki: Multiprint.

FERRÃO, M. (2010). "E-assessment within the Bologna paradigm: evidence from Portugal". *Assessment & Evaluation in Higher Education*. Vol. 35, No 7, pages 819-830.

GRAFF, M. (2003). "Cognitive Style and Attitudes Towards Using Online Learning and Assessment Methods". *Electronic Journal of e-Learning*, Vol. 1, No 1, pages 21-28.

HECK, A.; VAN GASTEL. L. (2006). "Mathematics on the threshold". *International Journal of Mathematical Education in Science and Technology*. Vol. 37, No 8, pages 925-945.

LIM, L. L.; THIEL, D. V.; SEARLES, D. J. (2011). "Fine tuning the teaching methods used for second year university mathematics". *International Journal of Mathematical Education in Science and Technology*. Pages 1-9, iFirst.

SMITH, G. H.; WOOD, L. N.; COUPLAND, M.; STEPHENSON, B.; CRAWFORD, K.; BALL, G. (1996). "Constructing mathematical examinations to assess a range of knowledge and skills". *International Journal for Mathematical Education in Science and Technology*. Vol. 27, No 1, pages 65-77.

STEEGMANN, C.; HUERTAS, M. A.; JUAN, A. A.; PRAT, M. (2008). "E-learning de las asignaturas del ámbito matemático-estadístico en las universidades españolas: oportunidades, retos, estado actual y tendencias / E-learning in the area of maths and statistics in Spanish universities: opportunities, challenges, current situation and trends". *RUSC*. Vol. 5, No 2, pages 1-14.

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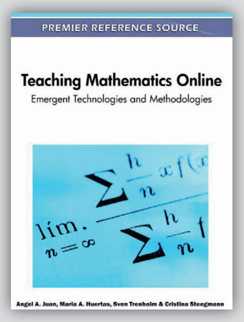
BOOK REVIEW



Teaching Mathematics Online: Emergent Technologies and Methodologies

Edited by Angel A. Juan, Maria A. Huertas, Sven Trenholm and Cristina Steegmann (2011).

Hershey, PA: IGI Global. 414 pages.

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Abstract

The following text reviews the book *Teaching Mathematics Online: Emergent Technologies and Methodologies*, recently published by IGI Global. This book brings together experiences and best practices related to the use of Web-based and computer-based methodologies to teach and learn mathematics courses in higher education. Although there is a plethora of books on e-learning and also a considerable amount of books on mathematics learning in secondary education, this is – as far as we know – the first book combining e-learning and mathematical education at the university level. Thus, it constitutes a basic reference for academics and practitioners of this constantly emerging field.

Keywords

e-learning, mathematical education, higher education, computer-supported learning

Resumen

El siguiente texto es una reseña del libro Teaching Mathematics Online: Emergent Technologies and Methodologies, publicado recientemente por IGI Global. En él se han reunido una serie de experiencias y mejores prácticas relacionadas con el uso de metodologías basadas en internet y en sistemas informáticos que tienen por objeto la enseñanza y el aprendizaje de las matemáticas en el ciclo educativo superior. Pese a la gran cantidad de libros existentes sobre e-learning y la abundancia de obras referidas a la enseñanza de las matemáticas en el ciclo educativo secundario, este es –que sepamos– el primer libro que combina e-learning y enseñanza de las matemáticas a un nivel universitario. Así pues, nos hallamos ante una referencia básica para entornos tanto académicos como profesionales de esta disciplina en constante evolución.

Palabras clave

e-learning, enseñanza de las matemáticas, enseñanza superior, aprendizaje asistido por ordenador

In this voluminous book of over 400 pages, the editors bring together 18 chapters on mathematics e-learning. They do so for two main reasons, as quoted below:

- "to provide insight and understanding into practical pedagogical and methodological issues related to mathematics e-learning," and
- "to provide insight and understanding into current and future trends regarding how mathematics instruction is being facilitated and leveraged with Web-based and other emerging technologies."

The book contains a variety of chapters, addressing many interesting developments within the area of technology-enhanced mathematics learning. It contains chapters discussing best practices regarding mathematics e-learning in higher education, chapters providing theoretical or applied pedagogical models in mathematics e-learning, chapters describing emerging technologies and mathematical software used in mathematics teaching online, as well as chapters presenting up-to-date research work on how mathematics education is changing through the use of online teaching methods.

The book starts with an introduction by the editors. They give an overview of the various chapters, which they have grouped into the following three sections:

1. Blended Experiences in Mathematics e-Learning
2. Pure Online Experiences in Mathematics e-Learning
3. Mathematics Software & Web Resources for Mathematics e-Learning

The chapters are equally divided over the three sections. We briefly summarize the content of the various sections and chapters.

The first section focuses on experiences in mathematical e-learning, in which face-to-face teaching is blended with distance or online instruction. It starts with a chapter by Miller describing the successful implementation of an asynchronous model for online discussions on a mathematics course for mathematics teachers. The section continues with a chapter by Abramovitz et al. on a blended experience in calculus courses for undergraduate engineering students, in which online assessments are used to help students understand theoretical concepts and theorems, and with a chapter by B. Loch, in which she describes how screencasts of live lectures as well as screencasts of short snippets of theory or examples have been used within an operations research course to supply online students with just-in-time information. Chapters 4 to 6 by Albano, Perdue and Divjak, respectively, discuss some experiences using general e-learning tools, ranging from LMSs, wikis and speaking avatars to video and social media, to enhance their face-to-face mathematics courses.

The second section of the book is devoted to experiences of purely online mathematics e-learning. It contains two chapters on the use of online communication and collaboration tools by Meletiou-Mavrotheris and by Silverman and Clay, both focusing on the education of mathematics teachers, and two chapters on the use and impact of online teaching material in bridging courses in mathematics for the transition from high school to university by Tempelaar et al. and by Biehler et al. The other two chapters by Jarvis and by Trenholm et al. both identify, review and evaluate a number of models and methods of mathematics e-learning.

The final section of the book is concerned with mathematical software and Web resources for mathematics e-learning. It contains a chapter by Cherkas and Welder reviewing some popular websites, a chapter by Alcazar et al. describing experiences with the software packages WIRIS, GeoGebra, SAGE and Wolfram Alpha, and a chapter by Lokar et al. describing the NAUK.si initiative to create Web-based learning blocks. Badger and Sangwin discuss the use of Gröbner basis techniques in the automatic grading of online exercises involving systems of equations. Misfeldt and Sanne discuss the problems that both students and lecturers face when writing mathematical formulas on a computer, as well as some solutions to these problems. The last chapter by Mac an Bhaird and O'Shea reviews a number of general-purpose software tools to be used in mathematics classes, including podcasts, screencasts and videos.

With this book, the editors have indeed succeeded in reaching their goals. They have brought together a great variety of interesting information about online Web resources and their use in both blended and online mathematics teaching. This collection of chapters provides a good insight into teaching methods, trends and possibilities offered by technology-enhanced mathematics learning.

Mathematics educators will certainly find both information and motivation in several chapters to improve their teaching through the good use of technology and online resources.

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